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A social network model of investment behaviour in the stock market

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ABSTRACT

To consider the psychological factors that impact market valuation, a model is formulated for investment behaviour of traders whose decisions are influenced by their trusted peers' behaviour. The model is implemented and several different "trust networks" are tested. Simulation results demonstrate that real life trust networks can significantly delay the stabilisation of a market.

1. Introduction

Under the efficient market hypothesis [1], prices of publicly traded assets (stocks, bonds, futures etc.) should reflect all the available relevant information. As a consequence, price should maintain a certain level of stability and only change when new information becomes available. Empirically, this hypothesis is contrary to the widely held ideas of "bull" and "bear" markets corresponding to the protracted periods of gradual increase or decrease of the asset prices in comparison to the benchmark trends (GDP, inflation, etc.). In particular, a significant challenge to the hypothesis comes from the existence of the extreme versions of "bull" and "bear" markets referred to as "market bubbles" and "market panics", defined as times with "trade in high volumes at prices that are considerably at variance with intrinsic values" [2–5]. This supports behavioural economics' claim that psychological biases prevent investors from acting fully rationally and thus undermine the basic premise of the efficient market hypothesis [6].

For example, in Ref. [7], the author divides the traders in the market into rational and emotional traders. Emotional traders always keep a bubble going whereas rational traders have the potential to burst the bubble. Conlon [8] considers the market as a two-player game in which each of the players follows the "law of greater idiots". Even if a player knows that the current stock is overpriced they may still buy, hoping that the other player will buy the stock at a higher price. In Ref. [9], Wei et al. propose that instability in the stock market is partly due to social influences impacting investors' decisions to buy, sell, or hold stock. By developing a Cellular Automata model of investment behaviour in the stock market they show that increased imitation among investors leads to a less stable market.

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In this paper we build on the basis laid by Ref. [9] and develop a social network model of the stock market. In particular, we follow [9]'s modelling technique and build a three-state (buy/sell/hold) model that takes into account a trader's peers. Traders' states are arrived at by considering: perceived price, change in price perception, the peers' influence, and 'personality'. The underlying premise of our model is that for a bubble to persist in a market composed of rational traders, those traders must have rational reasons to buy when prices are higher than stock value. These reasons – if they in fact exist – are almost certainly to be found in the behaviour of the other traders, as Ref. [9] pointed out. The behavioural influence, on the other hand, has to be tempered by the actual reality of the asset valuations. This can be viewed as a mix of information which is global (price), and information which is local to a node (trader) in the trust network. Interplay between stock value assessment and the behaviour of trusted traders is at the core of our model.

Our model also furthers the research of Wei et al., by extending the eight neighbour Cellular Automata structure used in Ref. [9, Eq. (2)] with an arbitrary (but static) network structure. This removes the unlikely assumption that each investor is influenced by exactly eight neighbours. Instead, each trader in the market has their own *trust network* (people they trust or distrust). The behaviour of trusted peers directly affects their behaviour, instead of indirectly and anonymously through price. It is important to note that the notion of "trust" in this context extends beyond simply believing that a trusted trader has better information about the actual value of the traded asset; it can also include the belief that the trusted trader's actions will broadly influence the market. The latter enables our model to incorporate the "social" effects of "superinvestors" such as large funds or highly influential individuals.

Naturally, our model also incorporates stock price into investor decisions, and investor decisions drive stock price. For example, if all investors seek to buy stock, then stock price will increase. Conversely, a high stock price will encourage investors to sell, driving price down. This provides a necessary feedback mechanism not found in Ref. [9]'s work. Finally, unlike Ref. [9], we assume that the amount of stock available is finite. Thus, for each trader who buys stock, there must exist a trader who sells stock. These notions provide an increased level of stability in the stock price, and investor behaviour. Nonetheless, our model demonstrates that social trust networks can cause highly stochastic behaviour in stock value and investor behaviour.

2. Social network model

The basic element of the model is the price of the good being traded. For the purposes of the model, price is normalised to zero around the long-term nominal value of the good. Long-term nominal value includes things such as risk estimates, rates of return, volatility, etc… A normalised price of zero means the price of the stock is equal to the value of the stock defined in this way. A positive normalised price means that the price of the good is higher than what the perfectly rational investor acting in isolation with a long-term investment goal should be willing to pay. This way of normalising price eliminates concerns about traders analyzing price history. Traders do so to determine whether a good is overpriced or underpriced. By our normalisation this "analysis" is assumed to be precise and available to all traders.

Like Ref. [9], the basic model consists of a collection of investors examined over a series of time steps, and we assume that at each time step investors exist in three possible states: buy, sell, and hold. Each trader is also given a list of other traders that they "trust". At each time step the model determines the normalised stock price. Each trader's decision to buy, sell, or hold is based on the current normalised price of the stock, the change in normalised price of the stock between the last time step and the current time step, and the actions of their trusted traders during the previous time step. The stock's normalised price is generated by determining what stock price nearest to the price at the previous time step would cause the number of traders who buy to be equal to the number of traders who sell.

In addition we study the effect of possible uncertainty about the true normalised price by replacing normalised price with a perceived normalised price. The perceived price is defined as the correct normalised price plus a small stochastic error based on a normal distribution around 0. Perceived prices are unique for each investor at each time step. A broad spectrum of trader knowledge scenarios and their effect on the market volatility can be studied using this model.

2.1. Notation and model parameters

We begin by setting our two indices:

- let $i \in I = \{1, 2, \ldots N\}$ represent the index of individuals within the model, and I be the set of all individuals; and
- let $t \in T = \{1, 2, \ldots\}$ represent time state, and T be the set of all possible time states.

In this paper, we will use a number of parameters that are specific to each trader, and some variables specific to each trader during a certain time step. Parameters are subscripted to indicate specific individuals and shown as a function of time when appropriate, e.g., A_i and $\varepsilon_i(t)$. When the subscript is omitted, the parameter or variable refers to the matrix (usually a column vector) in which the ith row 'belongs' to the ith individual.

For each pair of traders (i, j) we have

- $\alpha_{i,j}$ set to 0 or 1; a zero represents that trader i is not influenced by trader j, a 1 represents that trader i is influenced by trader j.

The collection of all $\alpha_{i,j}$ values forms the *trust network matrix* α, and α_i stands for the trust network of trader i.

For each individual trader i we have

- A_i, which defines how much the normalised asset price influences the individual. Traders with a high A_i parameter are more likely to buy undervalued assets and sell overvalued assets. Because the effect of all parameters is relative to the magnitude of the other parameters, we chose to normalise them such that $A_i = 1$ for all traders, as it is the only parameter that is non-negative within all reasonable scenarios.
- B_i, which defines how much the perceived change in normalised price influences the individual. Traders with a positive B_i see gains in normalised price as temporary events that will be followed by losses. Traders with a negative B_i believe in market trends (what goes up, will continue to go up).
- C_i, which defines how strongly the behaviour of others influences the individual. Traders with high C_i are imitators of the traders listed in their trust network. Traders with negative C_i distrust their peers and act in opposition to their actions.
- D_i, which defines an individual's 'innate' tendency towards buying or selling.[1]

In addition, each trader has their own perceived normalised price $p_i^p(t)$ defined as $p_i^p(t) = p(t) + \varepsilon_i(t)$ where $p(t)$ is the real normalised price as described earlier and $\varepsilon_i(t)$ is individual trader "error" which is normally distributed with a mean of 0 and a given standard deviation σ_ε. Scope for individual trader error can be studied by modifying σ_ε. By setting σ_ε to zero, we can test social effects in a perfect-information market. It should be noted that when $\sigma_\varepsilon = 0$, the model is fully deterministic.

2.2. Model rules

At time step $t - 1$ each individual i will have an action state $S_i(t) \in \{-1, 0, 1\}$. Here the state of -1 represents that the individual buys, the state of 1 represents that the individual sells, and the state of 0 represents that the individual holds. For ease of notation we define the state vector S at time t to be

$$S(t) = \begin{bmatrix} S_1(t) \\ S_2(t) \\ \vdots \\ S_N(t) \end{bmatrix}.$$

At the beginning of each time state we create a price error for each individual. That is, for each individual i we generate a personal skew factor $\varepsilon_i(t)$ from a normal distribution with mean of 0 and standard deviation of σ_ε. (The value σ_ε is an input and fixed at the outset of the model.)

The action state in time step t is determined by an individual leaning function $L_i(t)$. The leaning function is defined using the four personality parameters $A_i > 0$, $B_i \in \mathbb{R}$, $C_i \in \mathbb{R}$, and $D_i \in \mathbb{R}$, and the social influence vector α_i. These are fixed at the outset of the model. We defined the leaning function for individual i at time t as

$$L_i(t) = A_i(p(t) + \varepsilon_i(t)) + B_i(p(t) + \varepsilon_i(t) - (p(t-1) + \varepsilon_i(t-1))) + C_i\langle \alpha_i, S(t-1)\rangle + D_i$$
$$L_i(t) = A_i p_i^p(t) + B_i(p_i^p(t) - p_i^p(t-1)) + C_i\langle \alpha_i, S(t-1)\rangle + D_i$$

where $\langle \cdot, \cdot \rangle$ is the usual Euclidean inner product.

In order to determine the state of an individual we define a function F of the leaning as follows:

$$F(L) = \begin{cases} -1 & \text{if } L < b \\ 0 & \text{if } b \le L \le s \\ 1 & \text{if } s < L, \end{cases} \tag{1}$$

where $b \le 0 \le s$ are the buy–sell spread parameters ($b = -1$ and $s = 1$ for our model) representing thresholds of behaviour change. (Note that, as price increases, a trader's leaning will usually increase, causing the trader to stop buying stock or begin selling stock.) Using F, the state of individual i and time t is computed as $S_i(t) = F(L_i(t))$.

Therefore, individuals' action states at time t are based on information at time step $t - 1$ and price at time step t. Thus we are left with the question of how to define price at time step t.

In order to maintain conservation of stock within the market (recall we assume a finite and constant amount of asset) it is necessary that each buyer is matched with one seller (we assume without loss of generality[2] that all traders have equal buying power). Thus during each time state the stock value will fluctuate until the overall action state of the market has the same number of buyers as sellers. Mathematically, the price at the end of time state t will equalise to a point where

$$\sum_{i=1}^{n} F(L_i(t)) = 0.$$

[1] E.g., a brokerage firm may employ specialist 'buyers' and 'sellers'.

[2] A trader with increased buying power can be generated by cloning a particular trader and having the clones be influenced only by the 'main' trader and having a high C_i value. Partial investment could be simulated by carefully constructing a set of traders that under all conditions will have states in {buy, hold} or {sell, hold} and start buying or selling at a certain price level so as to increase or decrease the partial investment.

Therefore the price at the end of time state t (i.e. the beginning of time state $t+1$) should be the value $\tilde{p}$ such that

$$\sum_{i=1}^{N} F\big(A_i\tilde{p}_i^p(t) + B_i(\tilde{p}_i^p(t) - p_i^p(t-1)) + C_i\langle\alpha_i, S(t-1)\rangle + D_i\big) = 0 \tag{2}$$

where

$$\tilde{p}_i^p = \tilde{p} + \varepsilon_i(t).$$

Rewriting, we find

$$\sum_{i=1}^{N} F\big(A_i\tilde{p}_i^p(t) + B_i(\tilde{p}_i^p(t) - p_i^p(t-1)) + C_i\langle\alpha_i, S(t-1)\rangle + D_i\big)$$
$$= \sum_{i=1}^{N} F\big((A_i + B_i)\tilde{p}(t) + (A_i + B_i)\varepsilon_i(t) - B_i p_i^p(t-1) + C_i\langle\alpha_i, S(t-1)\rangle + D_i\big).$$

Let the constant $K_i(t) = (A_i + B_i)\varepsilon_i(t) - B_i p^p(t-1) + C_i\langle\alpha_i, S(t-1)\rangle + D_i$, to simplify Eq. (2) to

$$\sum_{i=1}^{N} F\big((A_i + B_i)\tilde{p}(t) + K_i(t)\big) = 0. \tag{3}$$

It is possible to show that as long as the quantities:

$$p_i^s = \frac{s - K_i}{A_i + B_i} \quad \text{and} \quad p_i^b = \frac{b - K_i}{A_i + B_i}$$

are unique to each trader, then Eq. (3) will have a solution. Indeed, solutions to Eq. (3) occur at any point p where the number of values p_i^b larger than p is equal to the number of values p_i^s smaller than p. In particular, one finds that

$$\sum_{i=1}^{N} F\big((A_i + B_i)p + K_i\big) = |\{k : p_k^b > p\}| - |\{j : p_j^s < p, \}|, \quad k, j = 1, 2, \ldots, N,$$

where $|\{\cdot\}|$ is the number of elements in the set $\{\cdot\}$. With this in mind, it is easy to locate a solution to Eq. (3) by ordering the set of all p_i^b and p_i^s values and applying a linear search of the midpoints between any two consecutive values. In the case of multiple solutions, we take the solution closest to the previous price.

In summary the model proceeds as follows:

0. Given: $A_i, B_i, C_i, D_i, \alpha, p(t-1), \varepsilon_i(t-1)$, and $S_i(t-1)$.
1. For each individual i randomly generate $\varepsilon_i(t)$.
2. Solve Eq. (3) to determine $p(t)$.
3. Apply $S_i(t) = F(L_i(t))$ to determine each individuals state at time t.
4. Increment t and repeat from 1 as desired.

3. Simulation and analysis

The driving feature of the Social Network Model developed here is the structure of the trust network matrix α. In our numerical experiments we test three different structures for the trust network, and show how model stability changes with network structure, both when the model runs deterministically ($\sigma_\varepsilon = 0$) and when a stochastic factor is included ($\sigma_\varepsilon = 0.33$). Our first test provides a baseline model output by setting the trust network to zero. That is, social influence is completely removed from the model. Our second test provides model output when the trust network is generated randomly using a uniform distribution. In our final test we use a real-world trust network developed by *Epinions* [10].

Model stability is measured in terms of price fluctuations, quantified by the Sum of Squares Error (SSE) of the normalised price over the last 25 time steps:

$$SSE = \sum_{t=26}^{50} (p(t) - \bar{p})^2,$$

where $\bar{p}$ is the mean price over the last 25 time steps of the model run. We only consider the second half of the time steps to allow plenty of time for the model to stabilise. Fig. 1 show that 25 time steps are more than sufficient; the efficient market stabilises within two time steps. For each test scenario we run 50 simulations, and report, in Table 1, the 10th, 25th, 50th, 75th and 90th percentile scores of SSE over all simulations. Each simulation uses 8000 traders, and the values for B_i, C_i and D_i are randomly generated in the following manner:

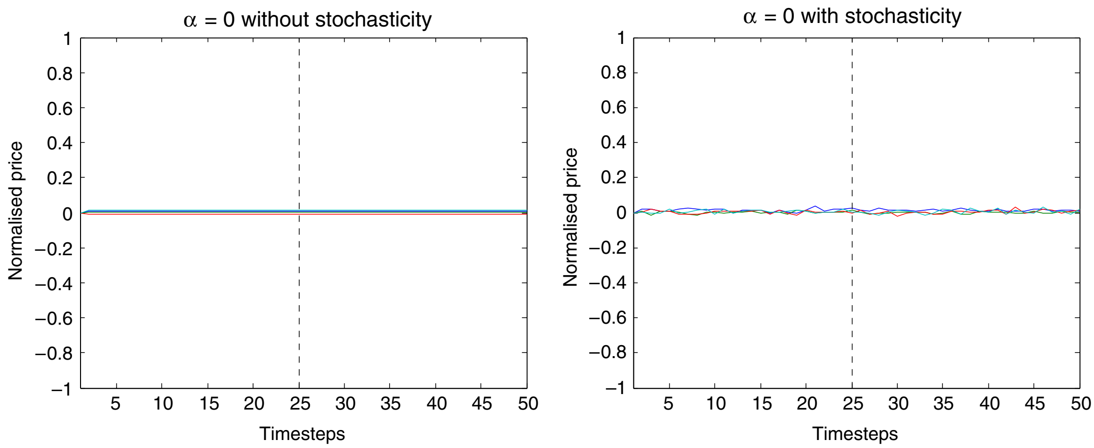

Fig. 1. Simulation visualization for $\alpha = \emptyset$, with $\sigma_\varepsilon = 0$ (left) and $\sigma_\varepsilon = 0.33$ (right), showing normalised market price from 4 simulation runs: the two with the lowest SSE (in green and dark blue), and the two with the highest SSE (in red and light blue). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 1
Percentile scores of the SSE for the tested scenarios, where $\sigma_\varepsilon = 0$ indicates the deterministic 'mode' and $\sigma_\varepsilon > 0$ indicates the stochastic 'mode'.

		10%	25%	50%	75%	90%
No trust	$(\sigma_\varepsilon = 0)$	$5.877 \cdot 10^{-35}$	$1.204 \cdot 10^{-33}$	$3.897 \cdot 10^{-18}$	$4.538 \cdot 10^{-7}$	$3.113 \cdot 10^{-6}$
	$(\sigma_\varepsilon > 0)$	$1.145 \cdot 10^{-3}$	$1.373 \cdot 10^{-3}$	$1.691 \cdot 10^{-3}$	$2.151 \cdot 10^{-3}$	$2.509 \cdot 10^{-3}$
Random trust	$(\sigma_\varepsilon = 0)$	$4.201 \cdot 10^{-4}$	$5.663 \cdot 10^{-4}$	$7.328 \cdot 10^{-4}$	$8.115 \cdot 10^{-4}$	$9.744 \cdot 10^{-4}$
	$(\sigma_\varepsilon > 0)$	$1.552 \cdot 10^{-3}$	$2.029 \cdot 10^{-3}$	$2.312 \cdot 10^{-3}$	$2.777 \cdot 10^{-3}$	$3.256 \cdot 10^{-3}$
Epinions	$(\sigma_\varepsilon = 0)$	$2.064 \cdot 10^{-5}$	$1.439 \cdot 10^{-4}$	$8.953 \cdot 10^{-4}$	$6.280 \cdot 10^{-3}$	$5.877 \cdot 10^{-2}$
	$(\sigma_\varepsilon > 0)$	$2.017 \cdot 10^{-2}$	$2.443 \cdot 10^{-2}$	$3.246 \cdot 10^{-2}$	$4.228 \cdot 10^{-2}$	$8.319 \cdot 10^{-2}$

- B_i is normally distributed around 0 with standard deviation 1;
- C_i is normally distributed around 5 with standard deviation 2;
- D_i is normally distributed around 0 with standard deviation 1;

The choice of these parameters follows in part Ref. [9]'s choice to investigate only scenarios where social influence is larger than all the other factors combined.

For the sake of visualization we plot the normalised market price from 4 simulation runs: the two with the lowest SSE (in green and dark blue), and the two with the highest SSE (in red and light blue).

3.1. Zero trust

In our first test we set $\alpha = \emptyset$. This effectively means $C_i = 0$ for each trader.

On the left hand side of Fig. 1 we see the result of the Efficient Market hypothesis: a stable market. On the right hand side of Fig. 1 we see how an actually considerable amount of uncertainty in price perception leads to only minor fluctuations in price in the stochastic equilibrium. It is likely that the fluctuations are this minor because the 8000 ε_i in each time state are drawn from a normal distribution around 0 and tend to cancel each other out (as said, $A_i = 1$ for all traders). This test provides a baseline for our further analysis.

3.2. Random sparse trust

In our second test we randomly generate the matrix α with a density approximately equal to that of the trust network used in Section 3.3 ($\approx 4.0 \cdot 10^{-3}$).

In Fig. 2 we see that introducing a randomly generated α still shows fast convergence to stochastic equilibrium. From Table 1 it can be seen that the stochastic model is still significantly different from the deterministic model. Comparing to the "No Trust" scenario, however, the runs with stochasticity are not significantly different. This is not entirely surprising when we consider that the randomly generated α *only* reproduces the density of the trust network used in Section 3.3, but not other structural characteristics. More specifically, the randomly generated α does not have the exponential degree distribution typical to social networks, but a normal degree distribution. This means that each trader has an approximately equal size random set of neighbours, about half of whom will expectedly have a higher B_i, C_i, and/or D_i, and about half will expectedly have a lower B_i, C_i, and/or D_i. This implies a level of global stability, although individual traders may still behave strangely.

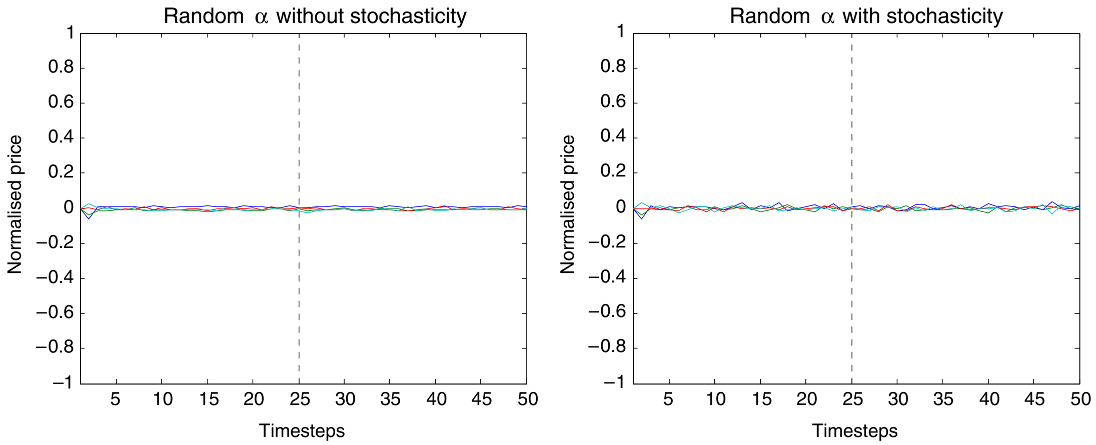

Fig. 2. Simulation visualization for randomly generated α, with $\sigma_\varepsilon = 0$ (left) and $\sigma_\varepsilon = 0.33$ (right), showing normalised market price from 4 simulation runs: the two with the lowest SSE (in green and dark blue), and the two with the highest SSE (in red and light blue). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

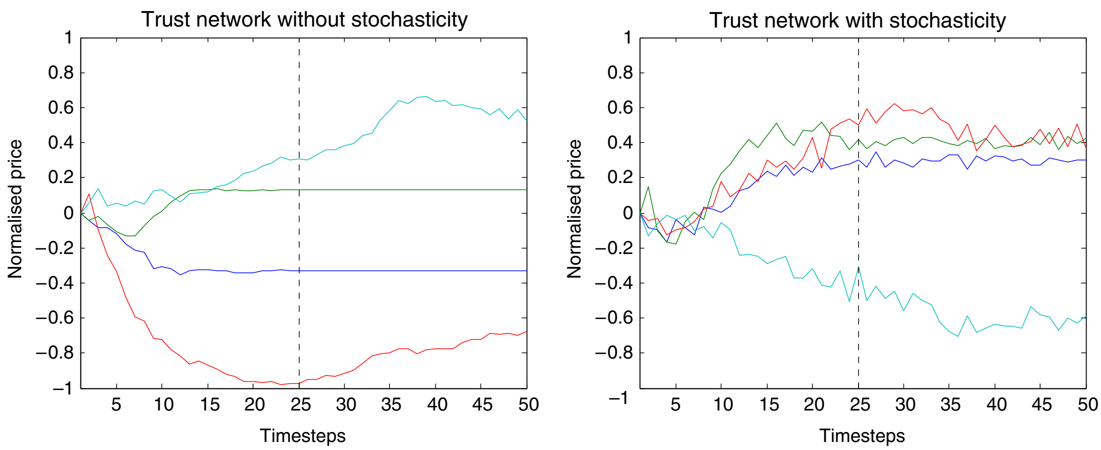

Fig. 3. Simulation visualization for the *Epinions* trust network as α, with $\sigma_\varepsilon = 0$ (left) and $\sigma_\varepsilon = 0.33$ (right), showing normalised market price from 4 simulation runs: the two with the lowest SSE (in green and dark blue), and the two with the highest SSE (in red and light blue). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

3.3. *Epinions trust network*

In our final test we generate α using the *Epinions* trust network data set. Epinions is a website where users write subjective reviews on different types of items [11]. The website has a broad range of items including software, music, hardware, and office appliances for purchase. Epinions is a trust-based recommender website where users are allowed to rate the usefulness of reviews. Users can also add other users to their "Web of trust" to see their reviews whenever they want to purchase an item. Since the Epinions data set represents trust in a market situation, it is reasonable to assume it is a good approximation of a realistic trust network distribution in an economical social network.

The data for the trust relationship between different users for the Epinions website is available and can be found at Ref. [10]. The data is provided in a two column table: the *source_user_id* and the *target_user_id* represent users by anonymised numeric identifiers. For example, the entry (1243, 2343) represents that user 1243 has expressed trust in user 2343 (but 2343 need not trust 1243). This format easily generates the matrix α, by setting $\alpha_{i,j} = 1$ whenever (i, j) appears in the data table. (All other entries in α are 0.)

In Fig. 3 we see that even in the deterministic case some runs of the model have stabilised and some have not stabilised. One likely explanation is that the degree distribution of the *Epinions* trust network is significantly different from the previous two cases. In particular, the links in the previous two cases were generated using a uniform distribution, leading to a normal degree distribution. This means all traders are equally influential. In contrast, analysis of the *Epinions* network shows that its degree distribution fits the following exponential distribution:

$$f_{in}(x) = 1891\mathrm{e}^{-0.3656x} \quad (r^2 = 0.9133),$$
$$f_{out}(x) = 421\mathrm{e}^{-0.0776x} \quad (r^2 = 0.9224),$$

where f_{in} is the degree distribution of traders being trusted by others and f_{out} is the degree distribution of traders trusting others. As a result, some traders are more influential than others. If these influential traders behave as the average trader would, the traders they influence will more likely display 'normal' behaviour. However, if a number of influential traders has a different strategy from the average trader they have the ability to create a complex social system that behaves in unexpected ways. For us, the most important observation here is that a trust network can greatly delay stability, regardless of stochasticity in trader uncertainty. (Stochastic equilibrium was almost instantaneous in the previous two scenarios.) Table 1 also tells us that the current scenario with stochasticity is significantly more variable than any of the previous scenarios, with or without stochasticity. This clearly demonstrates that a trust network has the capacity to exacerbate stochasticity.

4. Conclusion

In this paper we have advanced a Cellular Automata model of market behaviour using a 'customised' neighbourhood definition to show that the neighbourhood characteristics of real-world social networks can reduce stability in a system. As an example, we have simulated a market which is driven by price and social influence. When social influence is not included, our model displays the expected convergence to equilibrium that the efficient market hypothesis implies. Including social influence via a randomly generated neighbourhood only slightly decreases the stability of the system, but when a real-world trust network defines the neighbourhood, the results are markedly different. This shows that it is not simply the presence of social influence that will decrease stability, but rather the specific nature of the real-world trust network such as the Epinions network [10].

We believe there is much that is yet to be researched in this area. First, in this work we only very briefly examined the effect of trader uncertainty (ε) on model behaviour. Further analysis along those lines may lead to interesting results. Second, one could analyse which aspects of social networks are most closely related to (in)stability. This could mean analyzing how different scale-free networks affect model results. Third, we have not explored the impact of weighted social influence. It seems only natural that traders would trust one other trader more than another. In that light, it would perhaps be interesting to move towards a more agent-based simulation model in which one would allow traders to gradually change their opinions on their fellow traders, and thus allow α to evolve over time.

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