A1

(1) a. 3 candidate keys for R:  (A,B), (B,C), (B,D)  
   b. R is in 3NF, but not in BCNF.

(2) a. 4 candidate keys for S: A, B, C, D  
   b. S is in BCNF

(3) i. B is the candidate key and R is not in 3NF  
   ii. (A, B, C), (B, C, D) are candidate keys and R is in 3NF, but not in BCNF  
   iii. (A, B) is the key and R is not in 3NF

A2

1) a1, a2 -> a3, a4, a5  
   a3, a4 -> a1, a2, a5  

2) Yes, r1 is in BCNF because both {a1, a2} and {a3, a4} are the only non-trivial FDs, and both of them are superkeys.

3) Yes, r1 is in 3NF because if a relation is in BCNF, it is automatically in 3NF.
A3
(a) Minimum cover: \( A_1 \rightarrow A_3, A_3 \rightarrow A_4, A_3 \rightarrow A_6, A_3A_5 \rightarrow A_1, A_3A_5 \rightarrow A_2, A_5A_6 \rightarrow A_3. \)
(b) Only one: \( A_3A_5. \) Every candidate key must contain \( A_3A_5 \) (which don’t appear in RHS of FDs), and \( A_3A_5 \) is a key.
(c) FD1, FD2 and FD4 violate BCNF.
  - Start decomposing using FD4. We obtain \( R_1 = (A_1A_2, FD4), \) and \( R_2 = (A_1A_2A_4A_5A_6, \{FD1, FD2, FD3\}). \) \( R_1 \) is in BCNF, and \( R_2 \) is not in BCNF.
  - For \( R_2, FD1 \) and FD2 violate BCNF. Continue decomposing \( R_2, \) using FD2. We obtain \( R_3 = (A_1A_5, FD2) \) and \( R_4 = (A_3A_4A_5A_6, \{FD1, FD3\}). \) \( R_3 \) is in BCNF, and \( R_4 \) is not in BCNF.
  - For \( R_4, FD1 \) violates BCNF. Continue decomposing \( R_4, \) using FD1. We obtain \( R_5 = (A_3A_6, FD1), \) and \( R_6 = (A_3A_4A_5, FD3). \) Both \( R_5 \) and \( R_6 \) are in BCNF.

Thus, we obtained a dependency preserving BCNF decomposition because FD1 – FD4 have all of their attributes appearing in one of the relations (R1 – R6).

A4
1) I will focus only on the “Orders” relation and leave the other two relations for your practice.
   - order_no -> isbn, cid, card_no, qty, order_date, ship_date
   - cid -> card_no
   - isbn, cid, order_date -> order_no (let us assume that a customer only orders the same book at most once on any particular date)

2) Normalization for “Orders” relation. Again, I will leave the other two relations for your practice.

Orders (order_no, isbn, cid, qty, order_date, ship_date)
Card (cid, card_no)

It is easy to verify that Card is in BCNF. “Orders” is in BCNF because the two functional determinants are both candidate keys.