CPSC 259: Data Structures and Algorithms for Electrical Engineers

Hashing

Textbook Reference:

Thareja first edition: Chapter 15, pages 613-637
Thareja second edition: Chapter 15, pages 464-688

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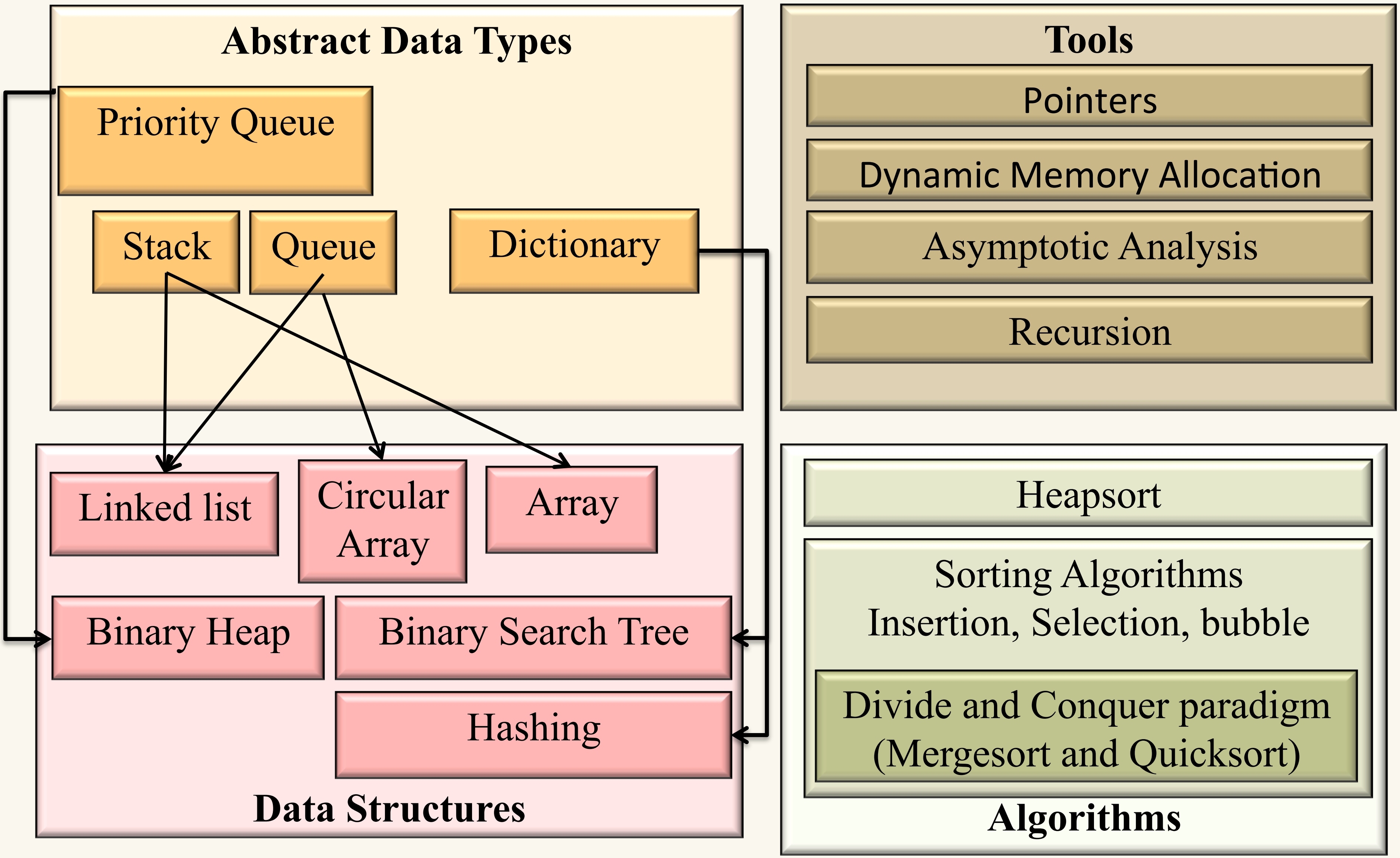
(Borrowing some slides from Steve Wolfman)

Learning Goals

After this unit, you should be able to:

- Define various forms of the pigeonhole principle; recognize and solve the specific types of counting and hashing problems to which they apply.
- Provide examples of the types of problems that can benefit from a hash data structure.
- Compare and contrast open addressing and chaining.
- Evaluate collision resolution policies.
- Describe the conditions under which hashing can degenerate from O(1) expected complexity to O(n).
- Identify the types of search problems that do not benefit from hashing (e.g. range searching) and explain why.
- Manipulate data in hash structures both irrespective of implementation and also within a given implementation.

CPSC 259 Journey



Reminder: Dictionary ADT

- Dictionary operations
  - create
  - destroy
  - insert
  - find
  - Delete

insert →

- brownies
  - tasty

← find(wolf)

- wolf
  - the perfect mix of oomph and Scrabble value

- midterm
  - would be tastier with brownies
- prog-project
  - so painful… who invented templates?
- wolf
  - the perfect mix of oomph and Scrabble value

- Stores values associated with user-specified keys
  - values may be any (homogenous) type
  - keys may be any (homogenous) comparable type

Implementations So Far

	insert	find	delete
• Unsorted list	O(1)	O(n)	O(n)
• Sorted Array	O(n)	O(log n)	O(n)
• BSTs	O(log n)	O(log n)	O(log n)

Can we do better? O(1)?

Example 1 (natural, numeric keys)

- In a small company of 100 employees, each employee is assigned an Emp_ID number in the range 0 – 99.
- To store the employee’s records in an array, each employee’s Emp_ID number acts as an index into the array where this employee’s record will be stored as shown in figure

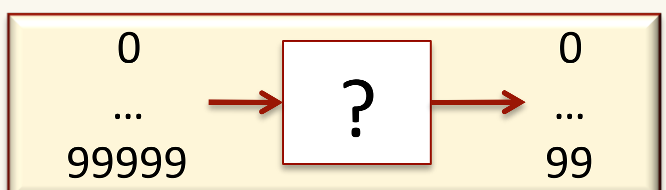
KEY		ARRAY OF EMPLOYEE’S RECORD
Key 0	**[0]**	**Record of employee having Emp_ID 0**
Key 1	**[1]**	**Record of employee having Emp_ID 1**
…………………………		………………………………………………..
Key 99	**[99]**	**Record of employee having Emp_ID 99**

Follow-up example

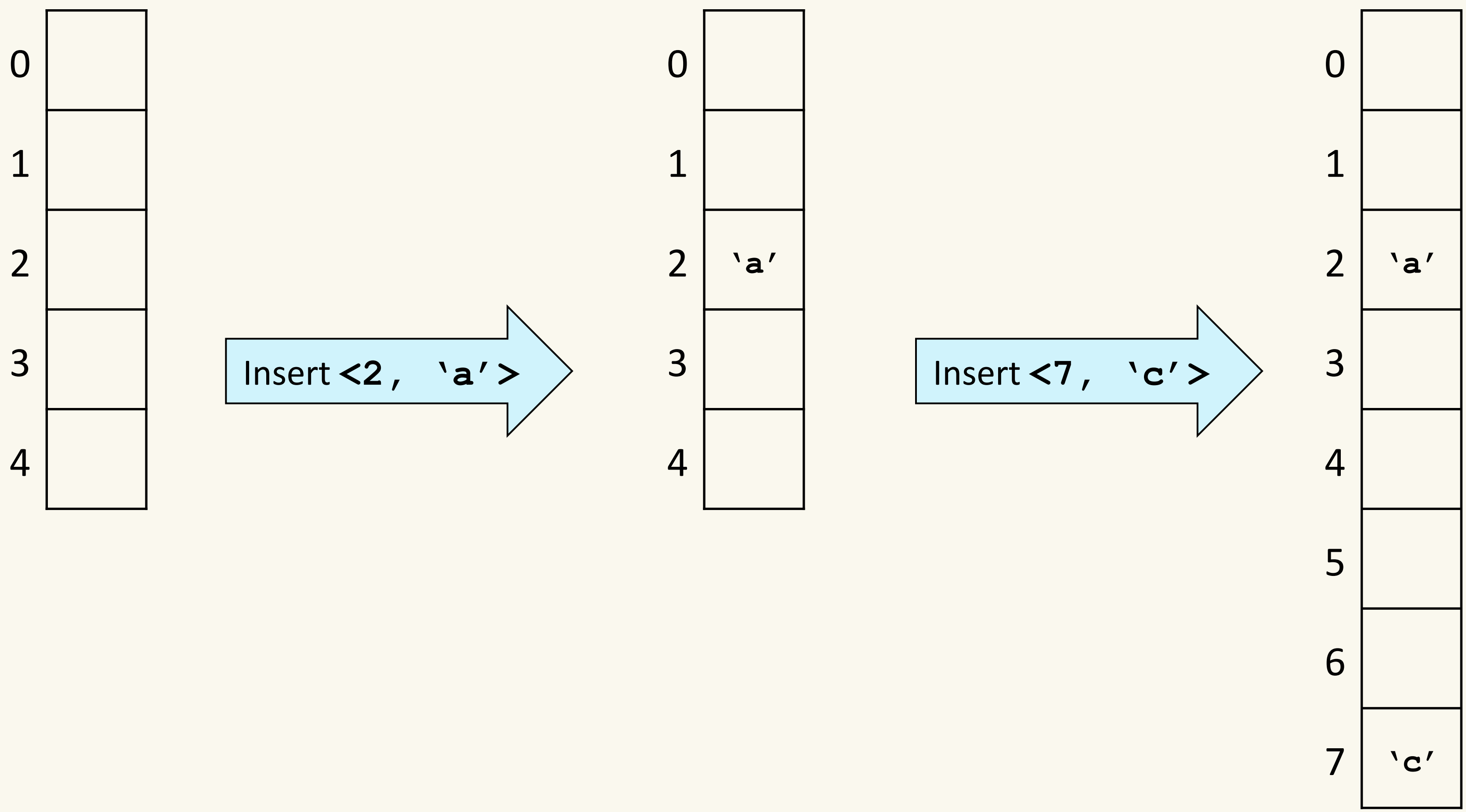
- Let's assume that the same company uses a five digit Emp_ID number as the primary key. If we want to use the same technique as above, we will need an array of size 100,000, of which only 100 elements will be used.

KEY	ARRAY OF EMPLOYEE' S RECORD
Key 00000 [0]	Record of employee having Emp_ID 00000
…………………………..	…………………………………………….
Key n [n]	Record of employee having Emp_ID n
…………………………..	……………………………………………..
Key 99999 [99999]	Record of employee having Emp_ID 99999

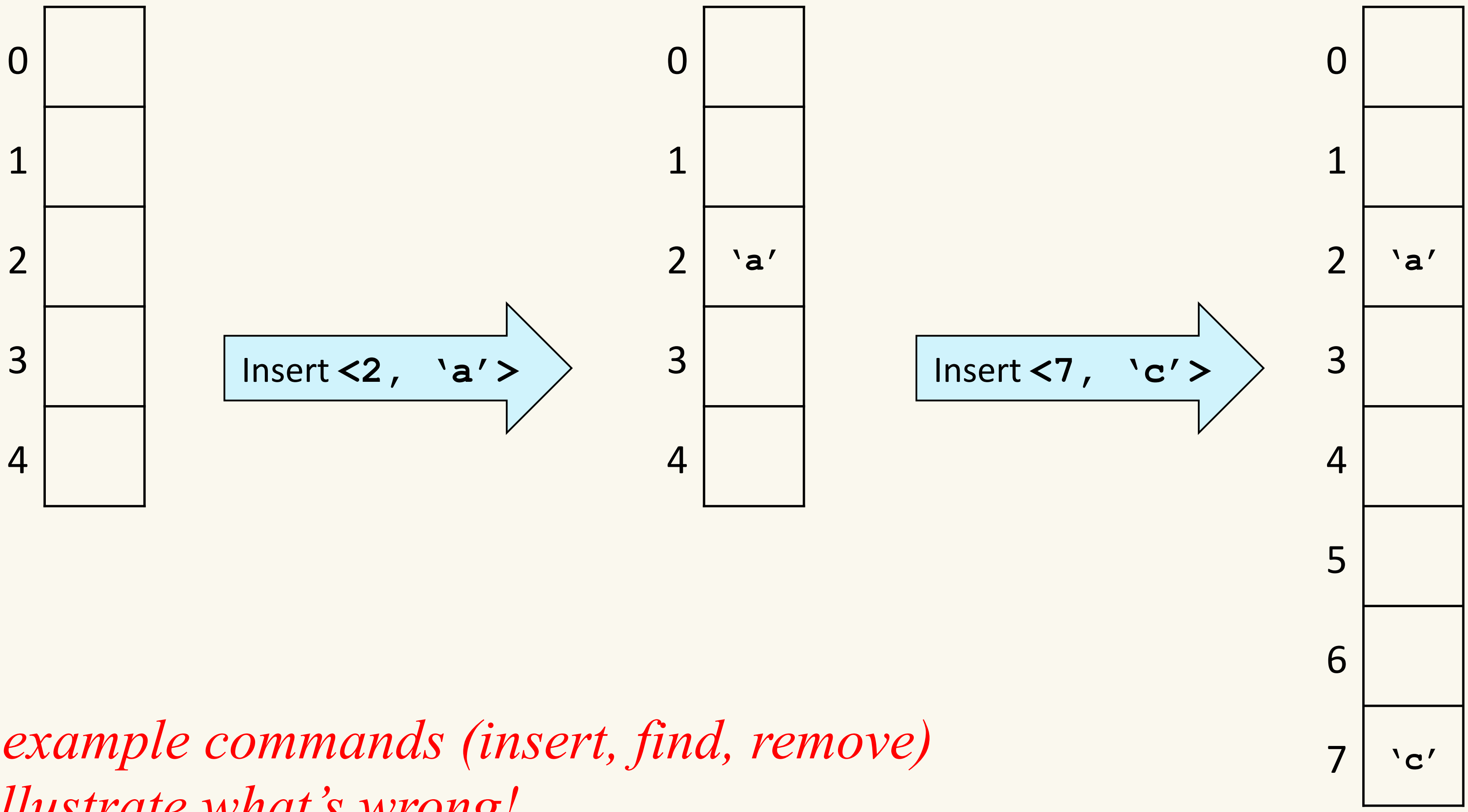
- It is impractical to waste that much storage just to ensure that each employee's record is in a unique and predictable location.



First Pass: Resizable Vectors



What's Wrong with Our First Pass?

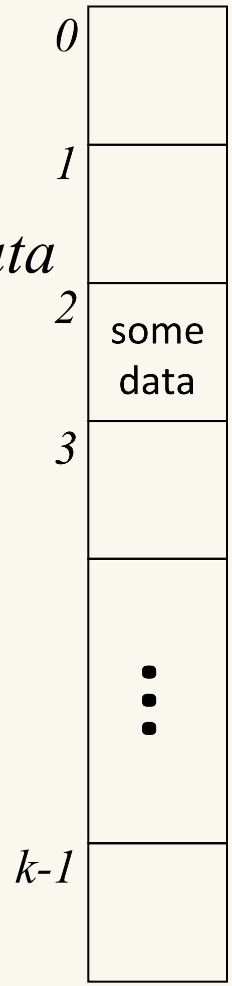


Give example commands (insert, find, remove) that illustrate what's wrong!

Hash Table Goal

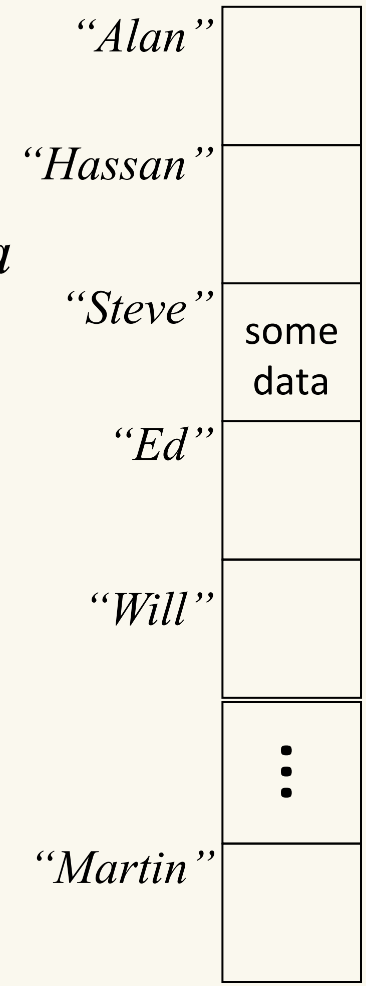
We can do:

a[2] = some data



We want to do:

a["Steve"] = some data

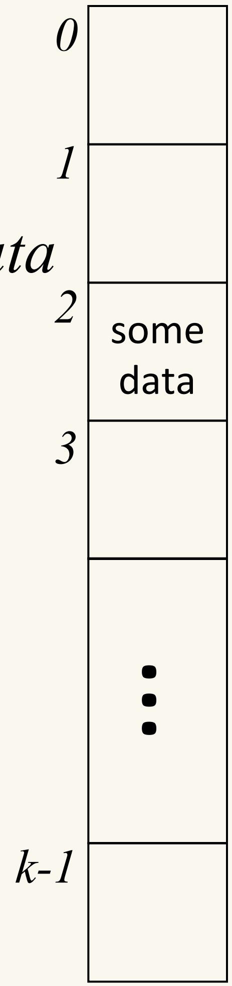


How will insert, find, and delete work?

Aside: How do arrays do that?

We can do:

a[2] = some data



Q: If I know houses on a certain block in Vancouver are on 33-foot-wide lots, where is the 5th house?
A: It's from (5-1)*33 to 5*33 feet from the start of the block.

element_type a[SIZE];

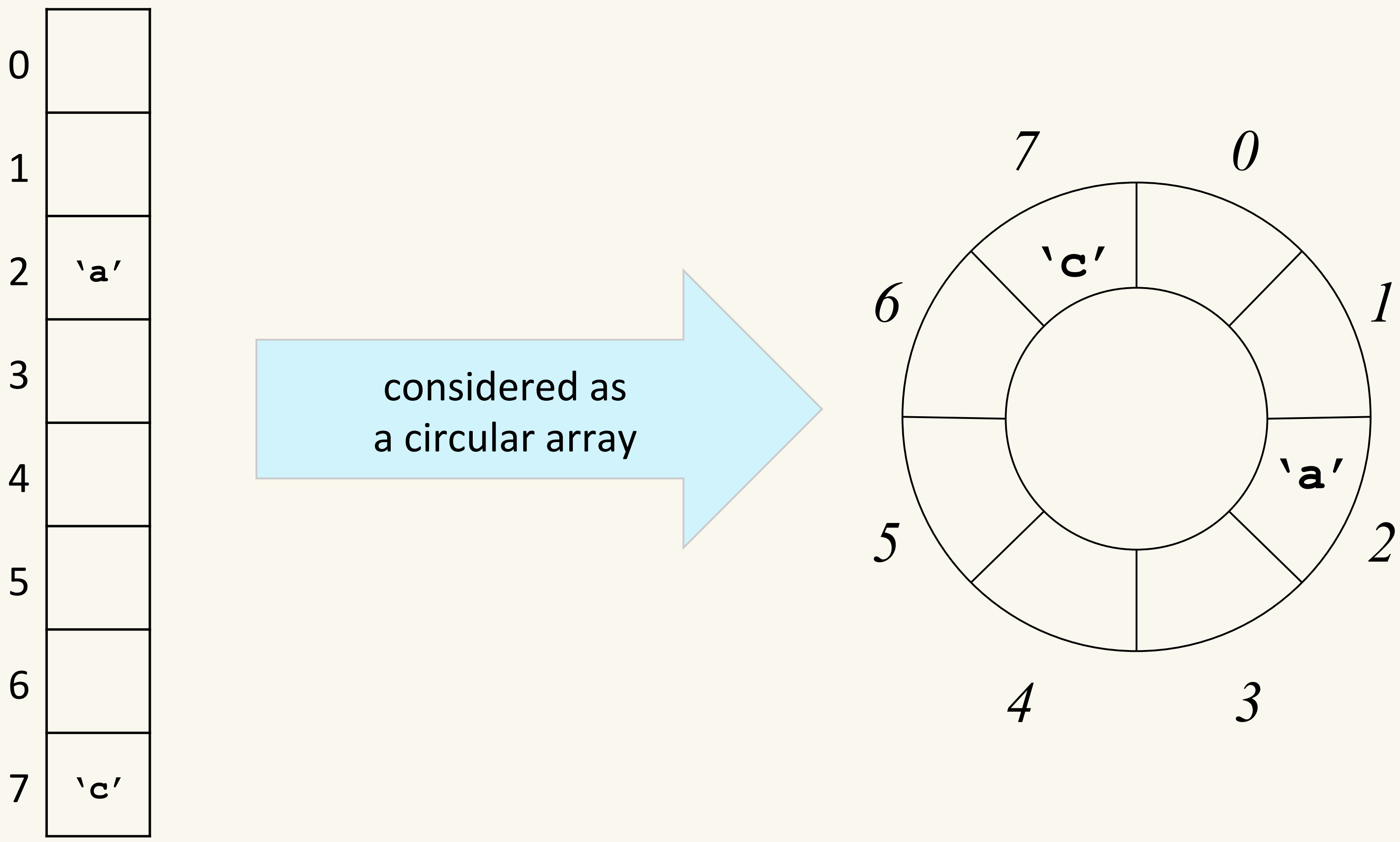
Q: Where is a[i]?
A: start of a + i*sizeof(element_type)

Aside: This is why array elements have to be the same size, and why we start the indices from 0.

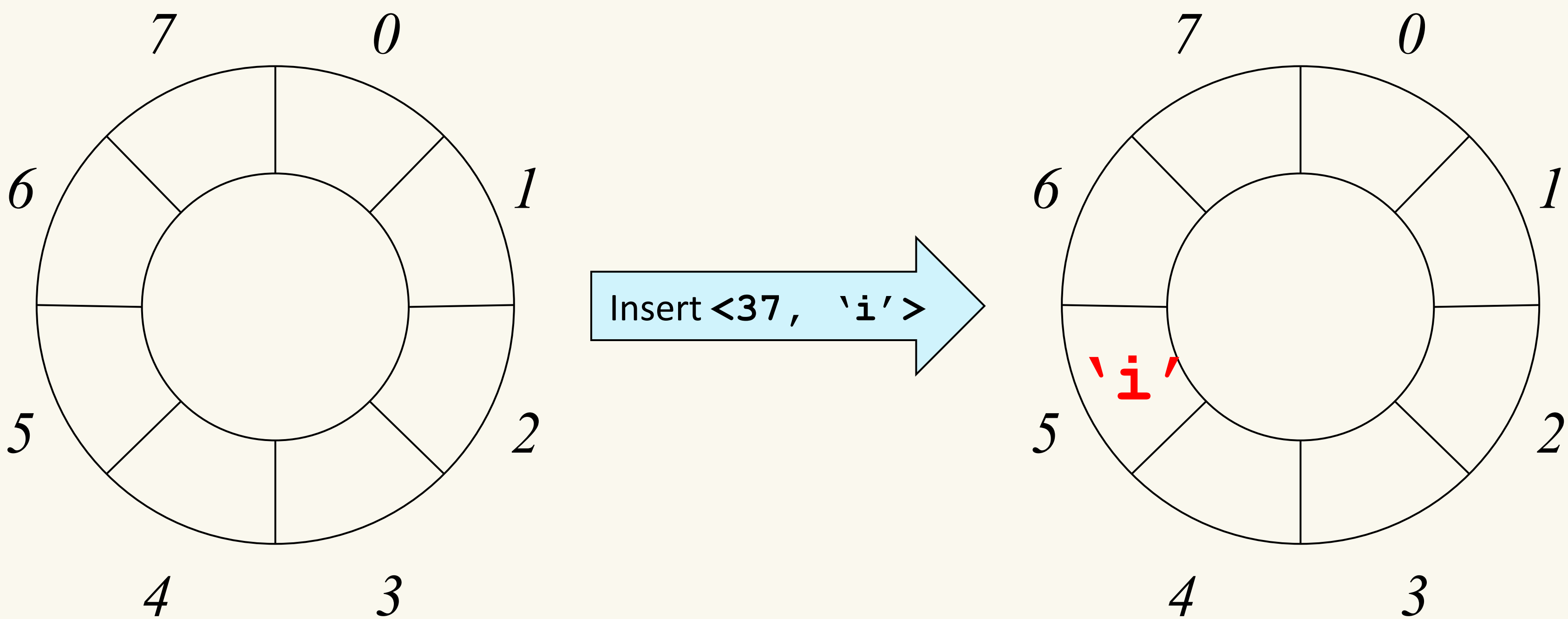
What is the 25th Element?

0	
1	
2	`'a'`
3	
4	
5	
6	
7	`'c'`

What is the 25th Element Now?

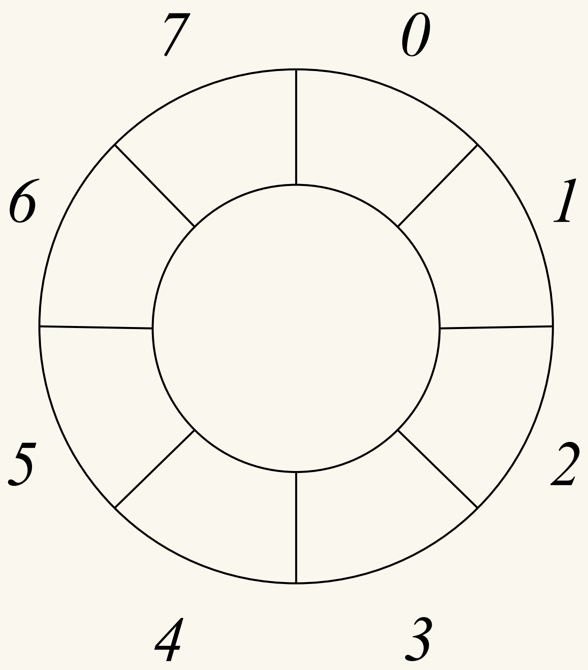


Second Pass: Circular Array (For the Win?)



Does this solve our memory usage problem?

What's Wrong with our **Second** Pass?



Let's insert 2 and 258?

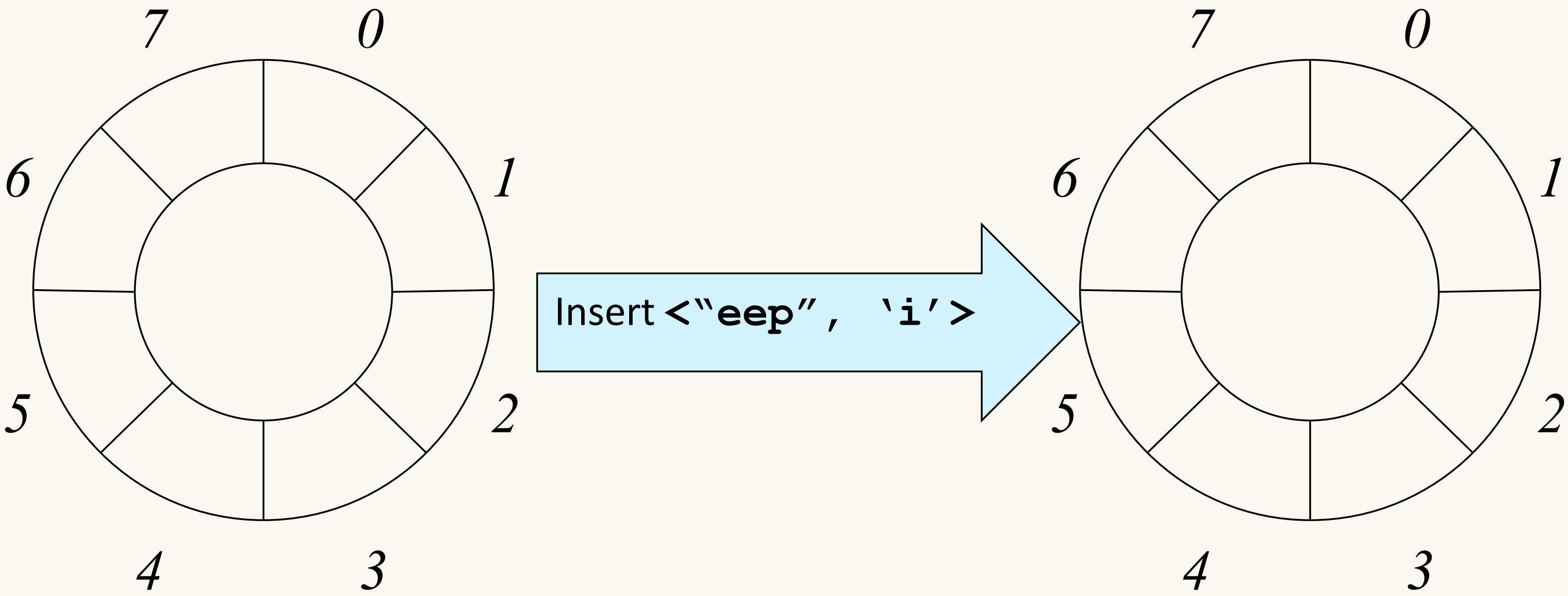
Resize until they don't?

258 % 8 = 2
258 % 16 = 2
258 % 32 = 2
258 % 64 = 2
258 % 128 = 2
258 % 256 = 2

Solutions:

- Prime table sizes helps
- Some way to handle these collisions without resizing?

How Do We Turn Strings into Numbers?

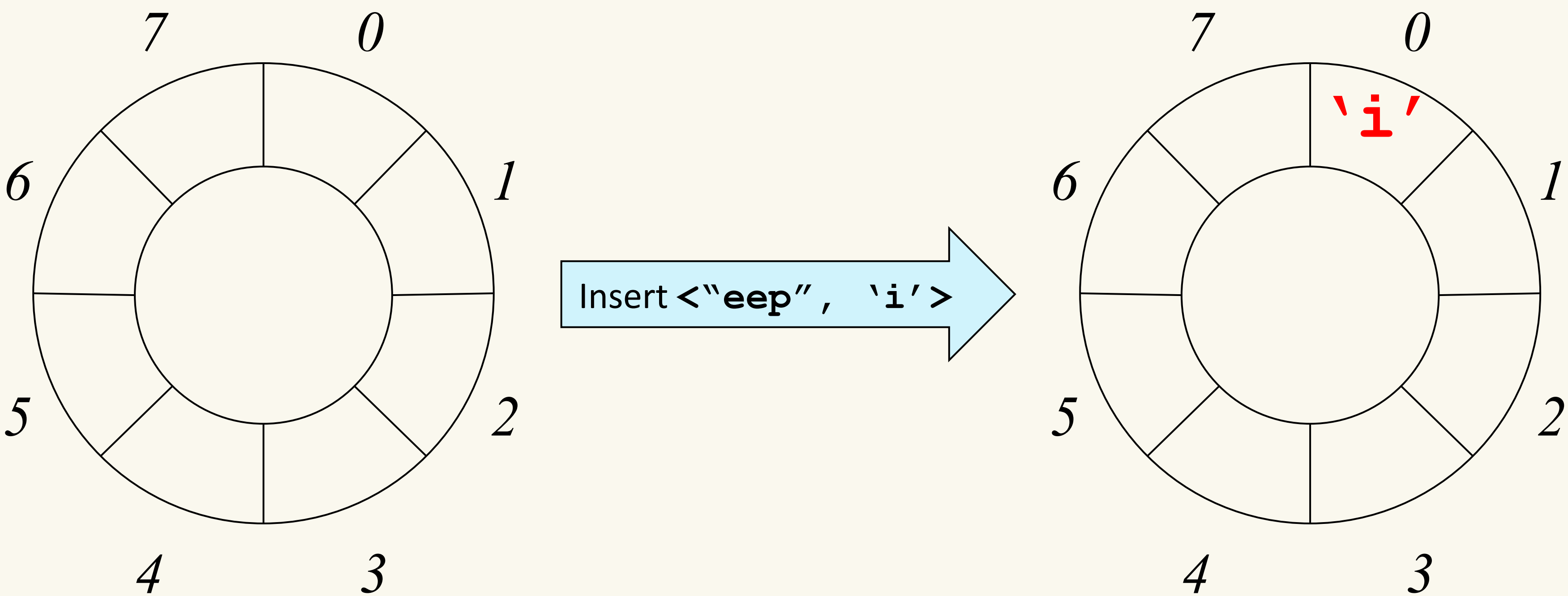


What should we do?

Third Pass: Strings **ARE** Numbers

e	e	p
01100101	01100101	01110000

$= 6{,}645{,}104$

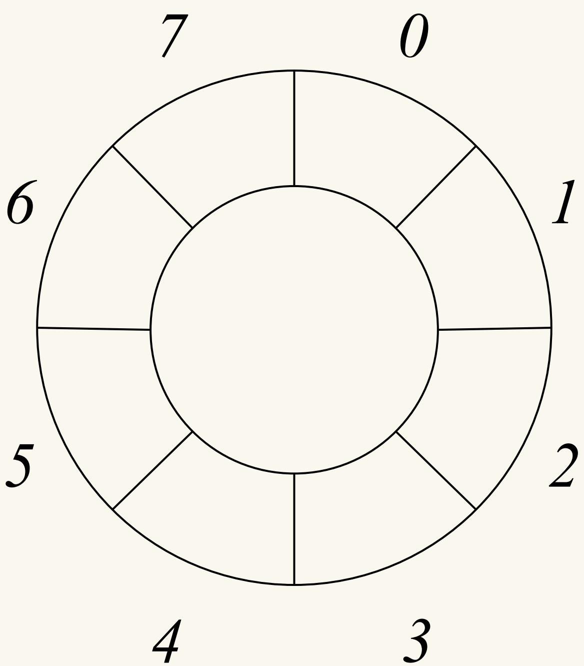


6,645,104 % 8 = 0

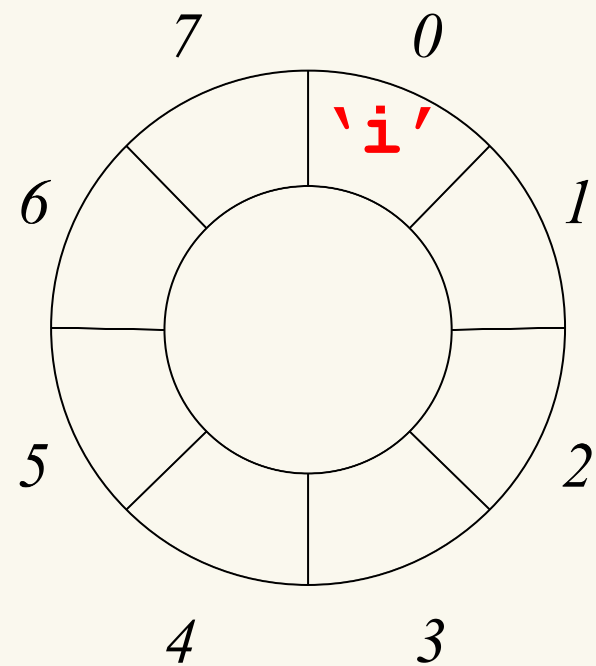
Third Pass: Strings **ARE** Numbers

e	e	p
01100101	01100101	01110000

= *6,645,104*



Insert <“**eep**”, ‘**i**’>



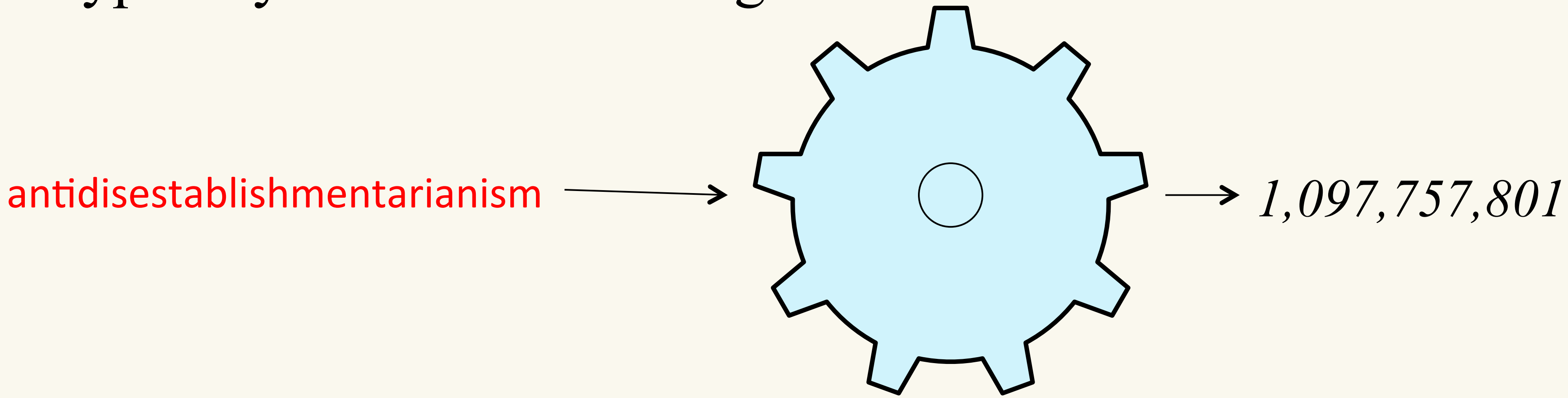
Those numbers get REALLY big antidisestablishmentarianism. Just saying.

6,645,104 % 8 = 0

Fourth Pass: Hashing!

- We only need perhaps a 64 (128?) bit number. There's no point in forming a **huge** number.

- We need a function to turn the strings into numbers, typically on a bounded range…



Maybe we can only use some parts of the string

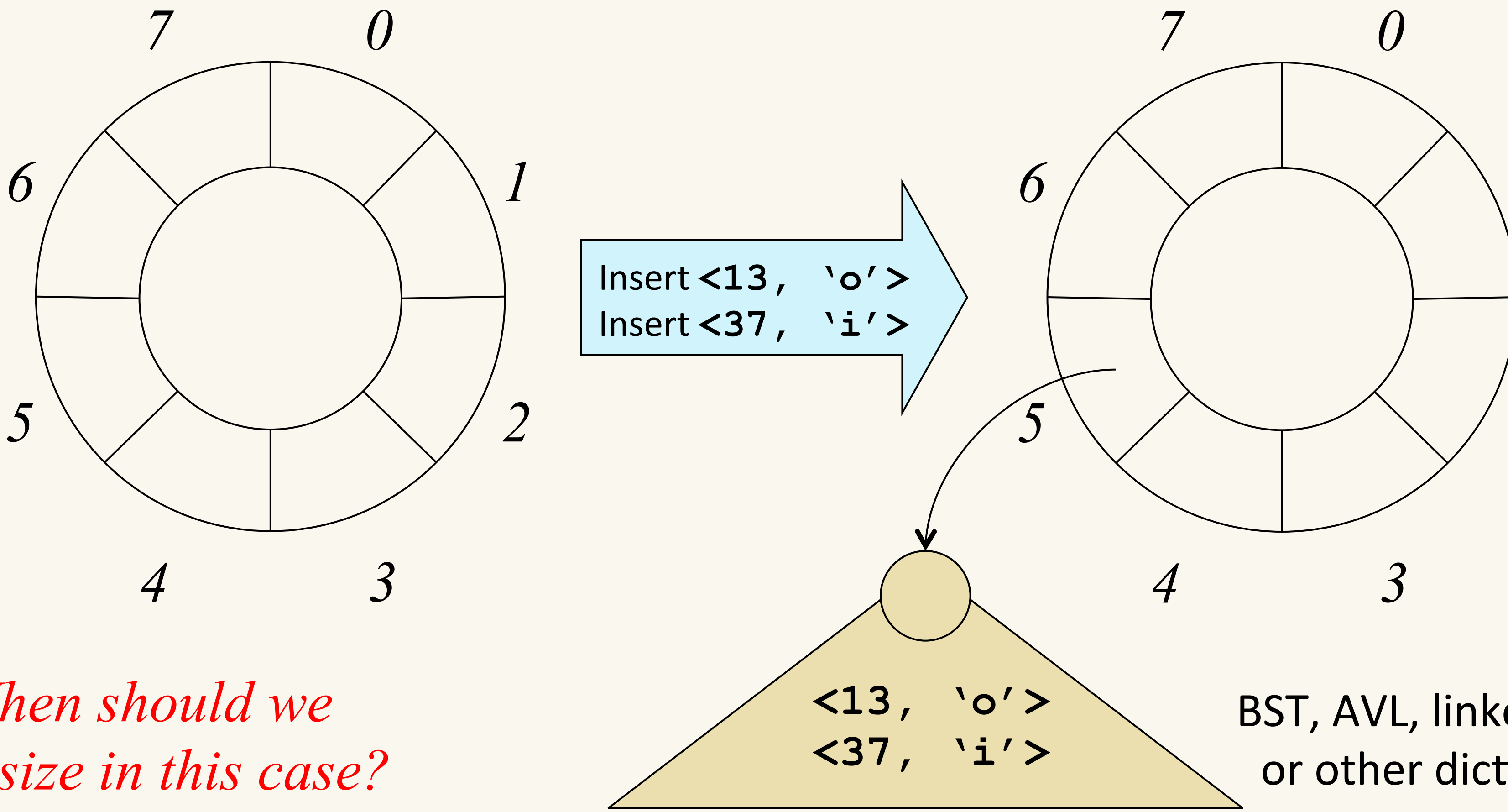
Schlemiel, Schlemazel, Trouble for Our Hash Table?

- Let’s try out:
  - “schlemiel” and “schlemazel”?
  - “microscopic” and “telescopic”?
  - “abcdefghijklmnopqrstuvwxyzyxwvutsrqponmlkjihgfedcba” and “abcdefghijklmnopqrstuvwxyzzyxwvutsrqponmlkjihgfedcba”

- Which bits of the string should we keep? Does our hash table care?

That’s hashing! Take our data and turn it into a sorta-random number, ideally one that spreads out similar strings far apart!

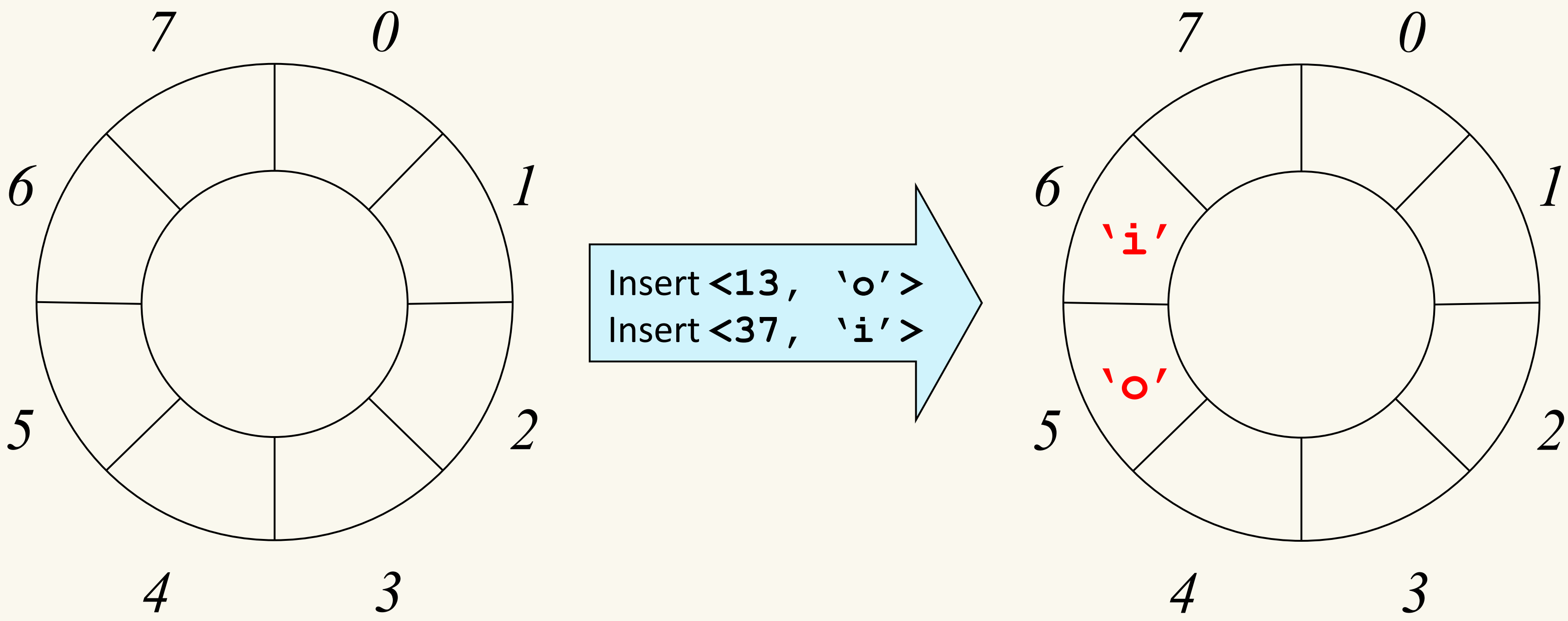
Punt to Another Dictionary?



When should we resize in this case?

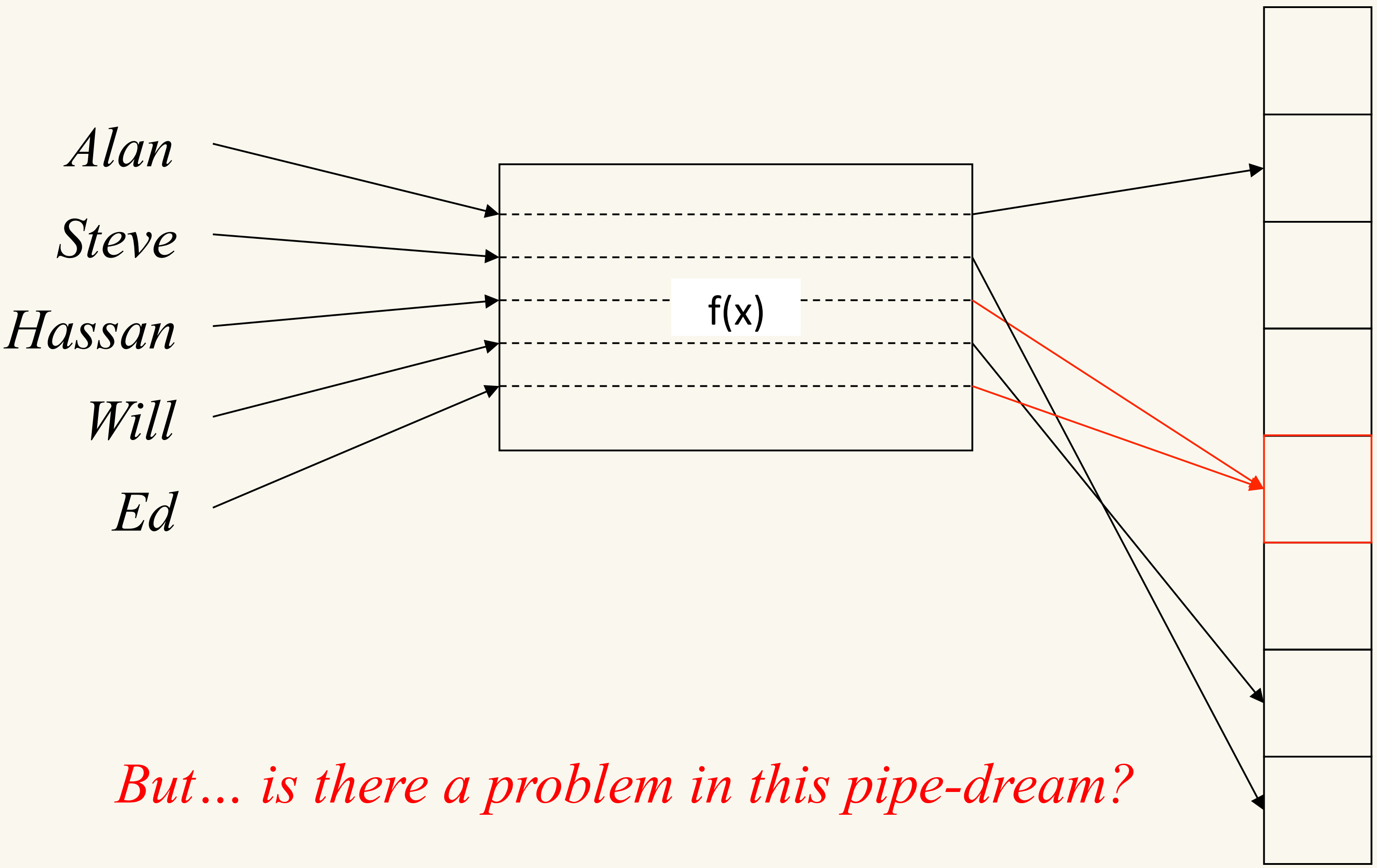
BST, AVL, linked list, or other dictionary

Punt to Another Slot?



Slot 5 is full, but no "dictionaries in each slot" this time.
Overflow to slot 6? When should we resize?

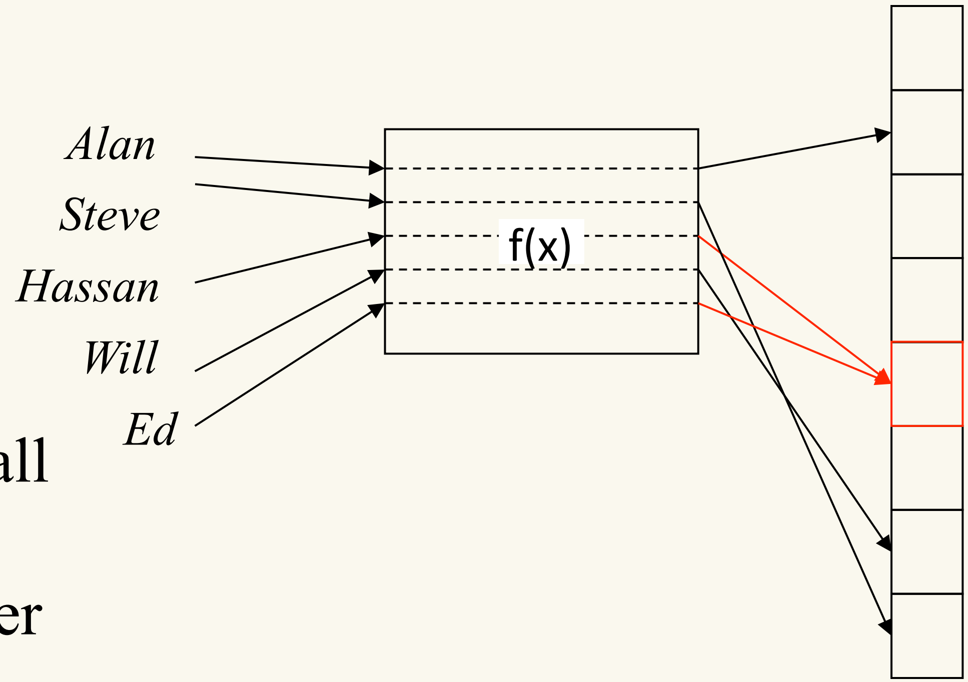
Hash Table Approach



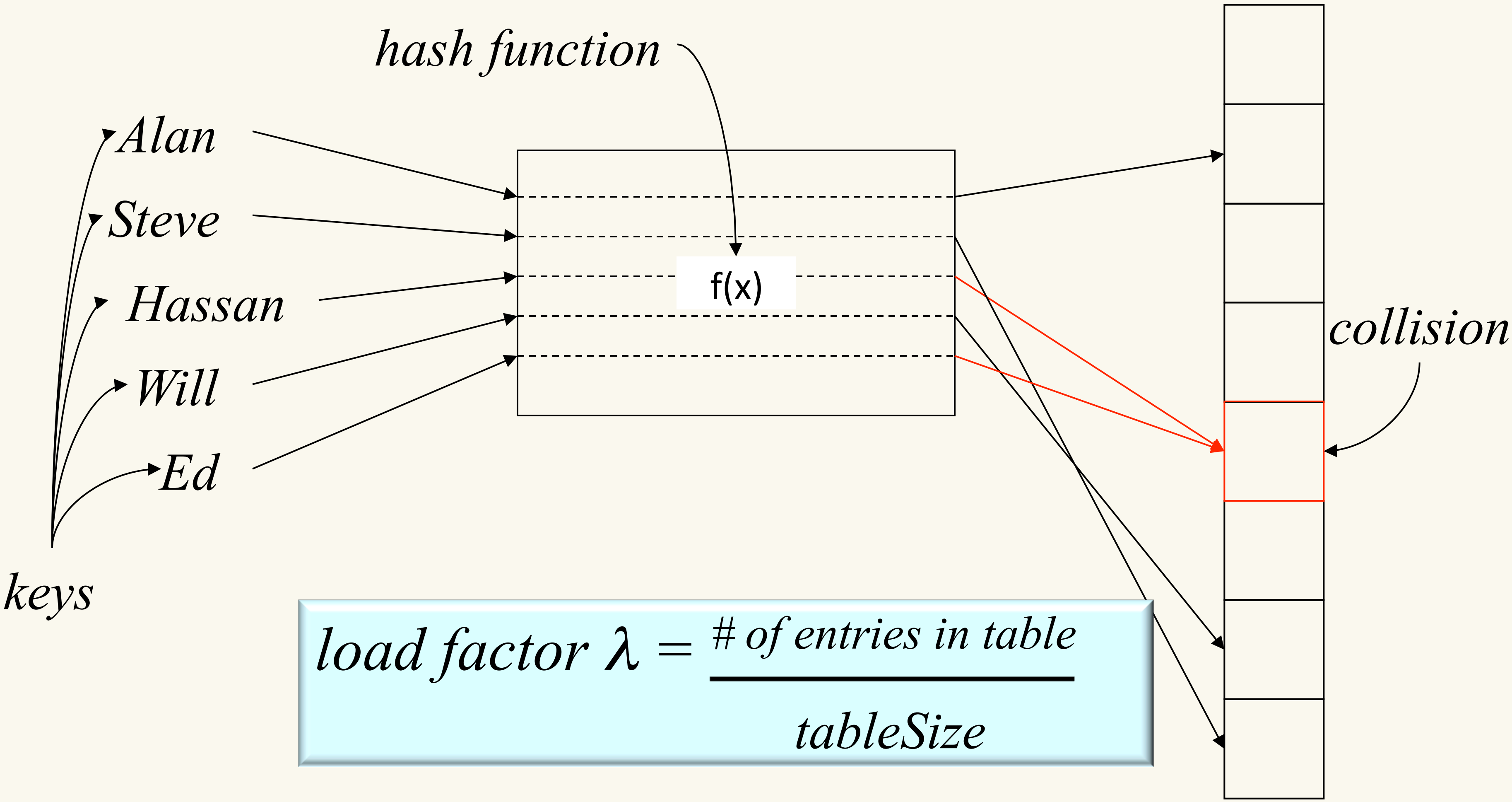
But… is there a problem in this pipe-dream?

Hash Table Dictionary Data Structure

- Hash function: maps keys to integers
  - result: can quickly find the right spot for a given entry
- Unordered and sparse table
  - result: cannot efficiently list all entries in order or list entries between one value and another (a "range" query)



Hash Table Terminology



$$\textit{load factor } \lambda = \frac{\#\textit{ of entries in table}}{\textit{tableSize}}$$

Hash Table Code First Pass

```
Value  find(Key  key) {
  int index = hash(key) % tableSize;
  return Table[index];
}
```

- What should the hash function be?
- What should the table size be?
- How should we resolve collisions?

A Good (Perfect?) Hash Function…

- is easy (fast) to compute
  - O(1) *and* fast in practice.
- distributes the data evenly
  - hash(a) % size ≠ hash(b) % size.
- uses the whole hash table. for all $0 \leq k < size$, there's an i such that
  - hash(i) % size = k.

Good Hash Function for Integers

- Choose
  - tableSize is
    - **prime** for good spread
    - **Resize using power of two** for fast calculations/convenient size
  - hash(n) = n % tableSize
    - (fast and good enough?)

Insert 2

Insert 5

Insert 10

Find 10

Insert 14

Insert -1

0	14
1	
2	2
3	10
4	
5	5
6	-1

Good Hash Function for Strings?

Suppose we have a table capable of holding 5000 records, and whose keys consist of strings that are 6 characters long. We can apply numeric operations to the ASCII codes of the characters in the string in order to determine a hash index:

```
int hash(char * key){
  int  hashCode = 0;
  int  index    = 0;
  while (key[index] != '\0'){
    hashCode += (int)key[index];
    index++;
  }
  return  hashCode % 5000;
}
```

C	A	M	E	C	O	/0

$$C = 67$$
$$A = 65$$
$$M = 77$$
$$E = 69$$
$$C = 67$$
$$O = 79$$
$$= 424$$

				CAMECO		

0   *424*   *4999*

Is this a good idea?

Good Hash Function for Strings?

- What is a significant problem with this approach?
  - Hash of any string with the same 6 letters is the same
  - ASCII values have a max of 255
    - 6*255 = 1530, which means [1531 – 4999] are wasted
- Alternative approach
- Let $s = s_1 s_2 s_3 s_4 \ldots s_5$: choose
  – $\text{hash}(s) = s_1 + s_2 128 + s_3 128^2 + s_4 128^3 + \ldots + s_n 128^n$
- Problems:
  – hash("really, really big") is really, really big!
  – hash("one thing") % 128 is close to hash("other thing") % 128

Making the String Hash Easy to Compute

- Use Horner's Rule (Qin's Rule?)

$$a + bx + cx^2 = a + x(b + xc)$$

```
int hash(string s) {
  h = 0;
  for (i = s.length() - 1; i >= 0; i--) {
    h = (s_i + 31*h) % tableSize;
  }
  return h;
}
```

hash(help) = h+31(e+31(l+31*p))

You would also need to %

Hash Function Summary

- Goals of a hash function
  - reproducible mapping from key to table entry
  - evenly distribute keys across the table
  - separate commonly occurring keys (neighbouring keys?)
  - complete quickly

How to Design a Hash Function

- Know what your keys are *or* Study how your keys are distributed.
- Try to include all important information in a key in the construction of its hash.
- Try to make “neighbouring” keys hash to very different places.
- Balance complexity/runtime of the hash function against spread of keys (very application dependent).

The Pigeonhole Principle (informal)

You can't put k+1 pigeons into k holes without putting two pigeons in the same hole.

This place just isn't coo anymore.

Image by en:User:McKay, used under CC attr/share-alike.

Clicker question

Suppose we have 5 colours of Halloween candy, and that there's lots of candy in a bag. How many pieces of candy do we have to pull out of the bag if we want to be sure to get 2 of the same colour?

a. 2
b. 4
c. 6
d. 8
e. None of these

Clicker question (answer)

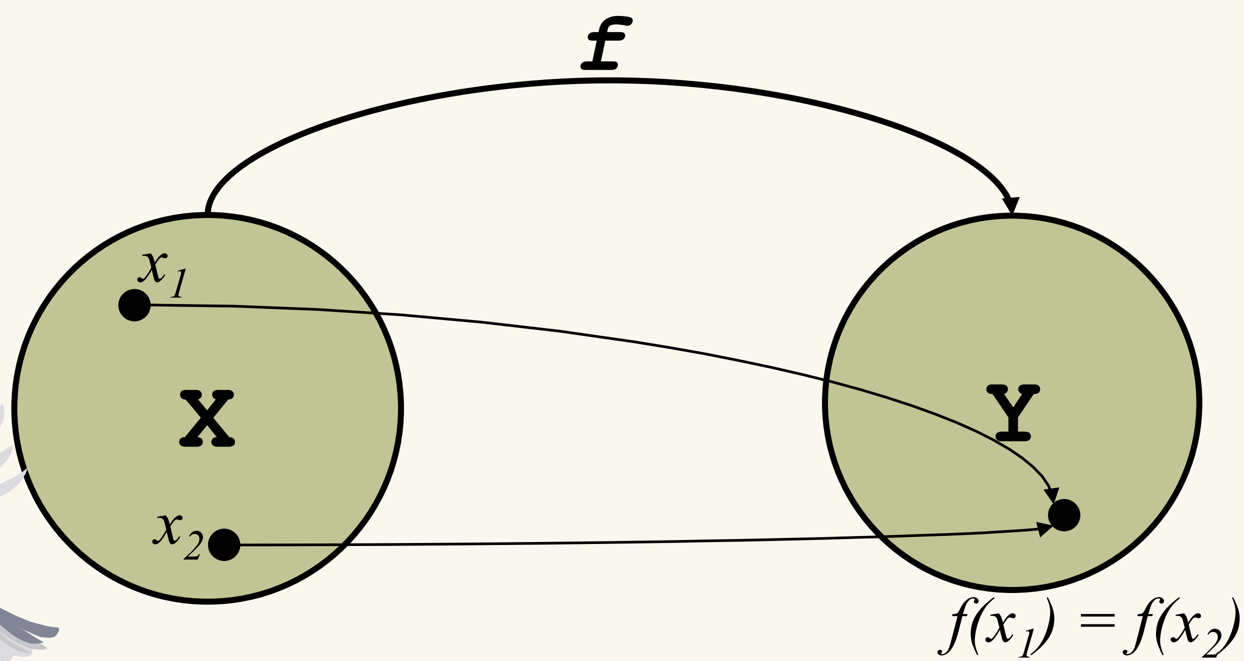
Suppose we have 5 colours of Halloween candy, and that there's lots of candy in a bag. How many pieces of candy do we have to pull out of the bag if we want to be sure to get 2 of the same colour?

a. 2
b. 4
c. 6
d. 8
e. None of these

The Pigeonhole Principle (formal)

Let X and Y be finite sets where |X| > |Y|.

If f : X→Y, then $f(x_1) = f(x_2)$ for some $x_1, x_2 \in X$, where $x_1 \neq x_2$.



The Pigeonhole Principle (Example #2)

If there are 1000 pieces of each color, how many do we need to pull to guarantee that we'll get 2 *black* pieces of candy (assuming that black is one of the 5 colors)?

a. 2
b. 6
c. 4002
d. 5001
e. None of these

The Pigeonhole Principle (Example #2)

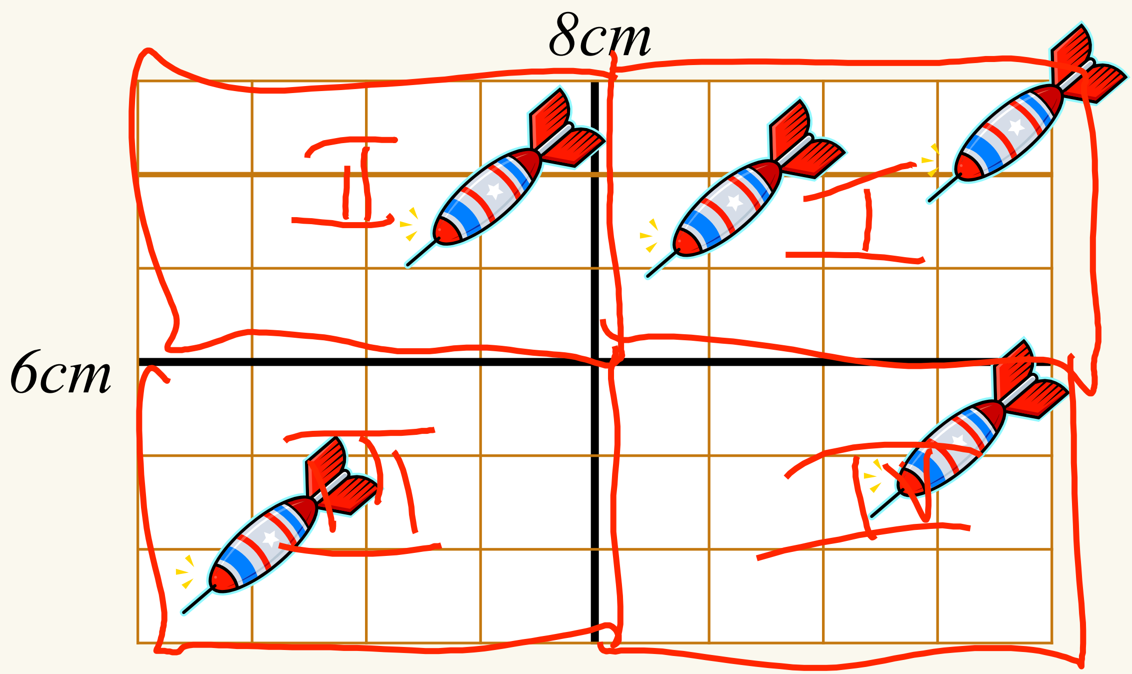
If there are 1000 pieces of each colour, how many do we need to pull to guarantee that we'll get 2 *black* pieces of candy (assuming that black is one of the 5 colours)?

a. 2
b. 6
c. 4002
d. 5001
e. None of these

This is not an appropriate problem for the pigeonhole principle! We don't know **which** hole has two pigeons!

The Pigeonhole Principle (Example #3)

If 5 points are placed in a 6cm x 8cm rectangle, argue that there are two points that are not more than 5 cm apart.



Hint: How long is the diagonal?

Example revisited

- In a small company of 100 employees, each employee is assigned an Emp_ID number in the range 00000 - 99999.
  - U (number of potential keys)=100,000
  - n (space allocated) =?
    - Hopefully not much bigger than m
    - Maybe 200 or 300
- By the Pigeonhole Principle(PHP) multiple potential keys are mapped to the same slot, which introduces the possibility of collisions.

Clicker question

- Consider n people with random birthdays (i.e., with each day of the year equally likely). How large does n need to be before there is at least a 50% chance that two people have the same birthday.

A: 23

B: 57

C: 184

D: 367

E: None of the above

Clicker question (Birthday Paradox)

- Consider n people with random birthdays. How large does n need to be before there is at least a 50% chance that two people have the same birthday.

A: 23 → 50%

B: 57 → 99%

C: 184

D: 367 → 100%

E: None of the above

- Corollary: Even if we randomly hash only $\sqrt{2m}$ keys into m slots, we get a collision with probability > **0.5**.

Collision Resolution

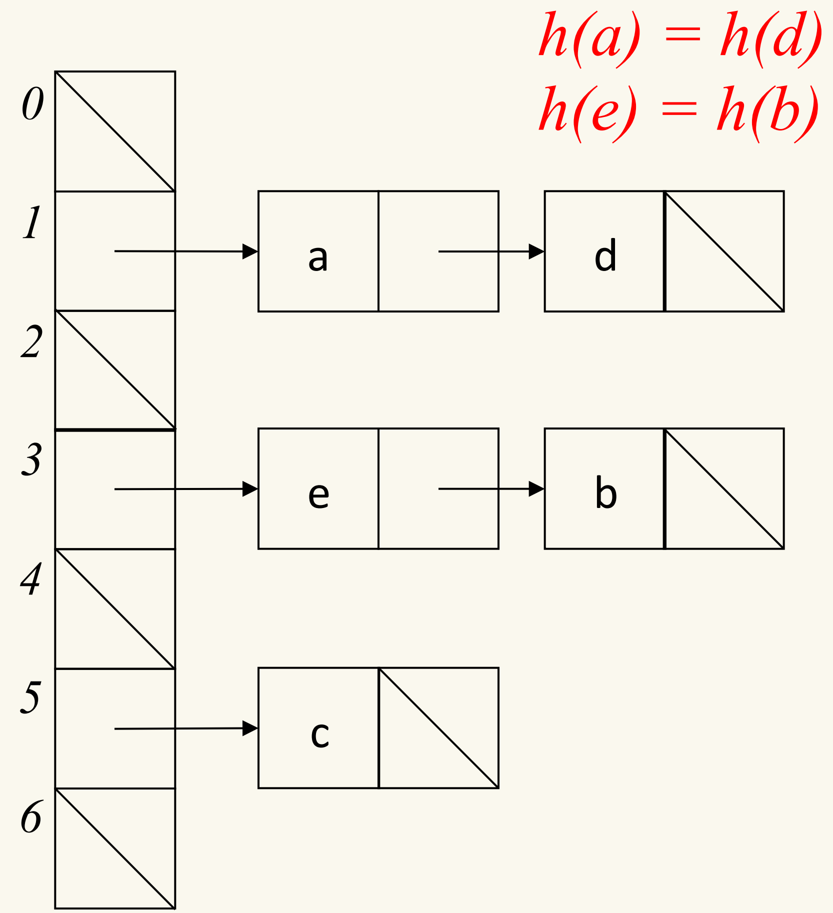
- What do we do when two keys hash to the same entry?
  - chaining: put little dictionaries in each entry

shove extra pigeons in one hole!

  - open addressing: pick a next entry to try

Hashing with Chaining

- Put a little dictionary at each entry
  - choose type as appropriate
  - common case is unordered move-to-front linked list (chain)
- Properties
  - λ can be greater than 1
  - performance degrades with length of chains

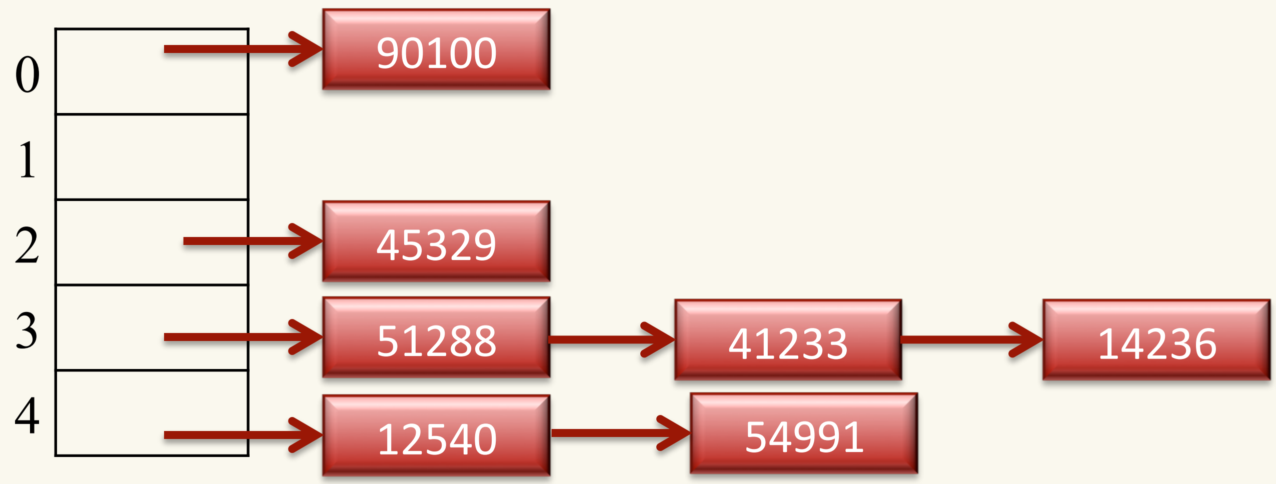


$$\text{load factor } \lambda = \frac{\#\text{ of entries in table}}{\text{tableSize}}$$

In-class exercise

Example: Suppose $h(x) = \lfloor x/10 \rfloor \bmod 5$

Hash: 12540, 51288, 90100, 41233, 54991, 45329, 14236

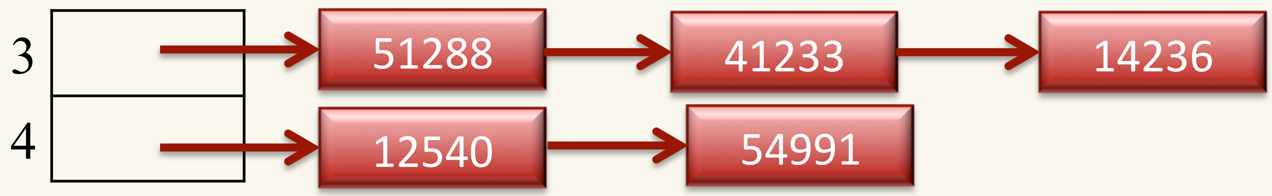


Example: find node with key 14236

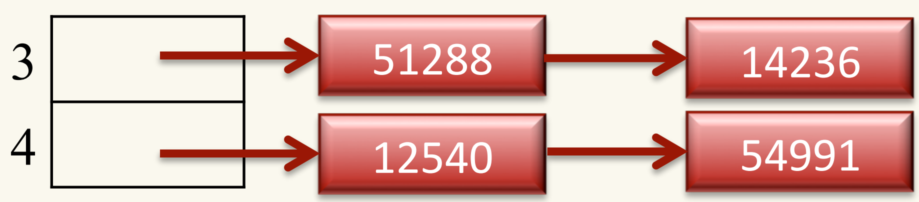
Deleting when using chaining

Example: Suppose $h(x) = \lfloor x/10 \rfloor \bmod 5$

Hash: 12540, 51288, 41233, 54991, 14236



- Delete 41233
- Remove 41233 from the linked list



Load Factor in Chaining

$$load\ factor\ \lambda = \frac{\#\ of\ entries\ in\ table}{tableSize}$$

- Search cost
  - unsuccessful search:
    - On average λ
  - successful search:
    - On average ~$\lambda/2$ +1 (what the book says)
    - More precisely

$$1+\frac{n-1}{2m} = 1+\frac{\lambda}{2}-\frac{\lambda}{2n}$$

(n-1)/m are
in this slot

- Desired load factor:
  - between 1/2 and 1.

Pros and cons of chaining

Advantages of Chaining:

- The size s of the hash table can be smaller than the number of items n hashed. Why is this often a good thing?
  - Fewer blank/wasted cells (especially in the case where the number of cells greatly exceeds the number of keys).
  - Collision handling can be O(1).
  - Can accommodate overflows

Disadvantages of Chaining:

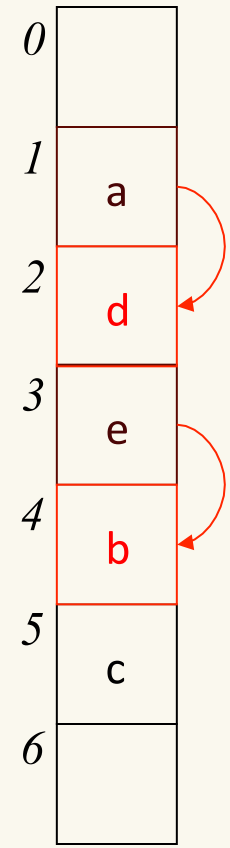
- Search time can become O(n) due to long chains.

Open Addressing

What if we only allow one Key at each entry?

- two objects that hash to the same spot can't both go there
- first one there gets the spot
- next one must *go in another spot*

$h(a) = h(d)$
$h(e) = h(b)$



- Properties
  - $\lambda \leq 1$
  - performance degrades with difficulty of finding right spot

$$\textit{load factor } \lambda = \frac{\#\textit{ of entries in table}}{\textit{tableSize}}$$

Probing

- Probing how to:
  - given a key k, hash to h(k)
  - if h(k) is occupied, try h(k) + f(1)
  - If h(k) + f(1) is occupied, try h(k) + f(2)
  - And so forth
- Probing properties
  - the i^{th} probe is to (h(k) + f(i)) mod size where f(0) = 0
  - if i reaches size, the insert has failed
  - depending on f(), the insert may fail sooner
  - long sequences of probes are costly!

Linear Probing, f(i) = i

- Probe sequence is
  - h(k) mod size
  - h(k) + 1 mod size
  - h(k) + 2 mod size
  - …

```
HASH-INSERT(T, k)
i = 0
repeat
    j = h(k, i)
    if T[j] == NIL
        T[j] = k
        return j
    else i = i + 1
until i == m
error "hash table overflow"
```

```
HASH-SEARCH(T, k)
i = 0
repeat
    j = h(k, i)
    if T[j] == k
        return j
    i = i + 1
until T[j] == NIL or i == m
return NIL
```

Clicker Question

- Using the hash function $h(x) = x \% 7$ insert the following values using **linear probing**: *76, 93, 40, 47, 10, 55*

- *In what index would would 55 be stored?*

- A: 6
- B: 0
- C: 1
- D: 2
- E: None of the above

Clicker Question (answer)

- Using the hash function $h(x) = x \% 7$ insert the following values using **linear probing**: *76, 93, 40, 47, 10, 55*

	insert(76) $76\%7 = 6$	*insert(93)* $93\%7 = 2$	*insert(40)* $40\%7 = 5$	*insert(47)* $47\%7 = 5$	*insert(10)* $10\%7 = 3$	*insert(55)* $55\%7 = 6$
0				47	47	47
1						55
2		93	93	93	93	93
3					10	10
4						
5			40	40	40	40
6	76	76	76	76	76	76
probes:	*1*	*1*	*1*	*3*	*1*	*3*

Load Factor in Linear Probing

$$\textit{load factor } \lambda = \frac{\textit{\# of entries in table}}{\textit{tableSize}}$$

- For *any* $\lambda < 1$, linear probing will find an empty slot
- Search cost (for large table sizes)
  - successful search: $\frac{1}{2}\left(1 + \frac{1}{(1-\lambda)}\right)$

	λ=0.25	λ=0.5	λ=0.75	λ=0.9
Avg # slots searched	1.17	1.5	2.5	5.5

  - unsuccessful search: $\frac{1}{2}\left(1 + \frac{1}{(1-\lambda)^2}\right)$
    - How performance degrades as λ gets bigger

	λ=0.25	λ=0.5	λ=0.75	λ=0.9
Avg # slots searched	1.4	2.5	8.5	50.5

Load Factor in Linear Probing

$$load\ factor\ \lambda = \frac{\#\ of\ entries\ in\ table}{tableSize}$$

- For *any* $\lambda < 1$, linear probing will find an empty slot
- Search cost (for large table sizes)
  - successful search: $\frac{1}{2}\left(1 + \frac{1}{(1-\lambda)}\right)$
  - unsuccessful search: $\frac{1}{2}\left(1 + \frac{1}{(1-\lambda)^2}\right)$

Values hashed close to each other probe the same slots.

- Linear probing suffers from ***primary clustering***
- Performance quickly degrades for $\lambda > 1/2$

Quadratic Probing, $f(i) = i^2$

- Probe sequence is
  - $h(k)$ mod size
  - $(h(k) + 1)$ mod size
  - $(h(k) + 4)$ mod size
  - $(h(k) + 9)$ mod size
  - …

Clicker Question

- Using the hash function $h(x) = x \% 7$ insert the following values using **quadratic probing**: *76, 40, 48, 5, 55*

- *In what index would would 55 be stored?*

- A: 6
- B: 0
- C: 1
- D: 3
- E: None of the above

Quadratic Probing Example ☺

- Using the hash function $h(x) = x \% 7$ insert the following values using **quadratic probing**: *76, 40, 48, 5, 55*

	insert(76)	*insert(40)*	*insert(48)*	*insert(5)*	*insert(55)*
	76%7 = 6	*40%7 = 5*	*48%7 = 6*	*5%7 = 5*	*55%7 = 6*
0			48	48	48
1					
2				5	5
3					55
4					
5		40	40	40	40
6	76	76	76	76	76
probes:	*1*	*1*	*2*	*3*	*3*

Clicker Question

- Using the hash function $h(x) = x \% 7$ insert the following values using quadratic probing: *76, 93, 40, 35, 47*
- *In what index would would 47 be stored?*
- A: 6
- B: 0
- C: 1
- D: 3
- E: None of the above

Quadratic Probing Example ☹

- Using the hash function $h(x) = x \% 7$ insert the following values using quadratic probing: *76, 93, 40, 35, 47*

	insert(76) $76\%7 = 6$	*insert(93)* $93\%7 = 2$	*insert(40)* $40\%7 = 5$	*insert(35)* $35\%7 = 0$	*insert(47)* $47\%7 = 5$
0				35	35
1					
2		93	93	93	93
3					
4					
5			40	40	40
6	76	76	76	76	76
probes:	*1*	*1*	*1*	*1*	∞

Quadratic Probing Succeeds (for λ ≤ ½)

- Claim: If size is prime, the first size/2 probes are distinct
- Proof : omitted
- Result: If size is prime and λ ≤ ½, then quadratic probing will find an empty slot in size/2 probes or fewer

- Quadratic probing does not suffer from primary clustering
- Quadratic probing *does* suffer from ***secondary* clustering**
  - How could we possibly solve this?

Values hashed to the SAME index probe the same slots.

CPSC 259 Administrative Notes

- Lab 5 take-home due Sun Dec 6
  - Feel free to drop by the second hour of other lab sections to get help.
- Connect quiz and textbook exercises on Hashing are now available

- Concept Inventory
  - A study to help us understand some of the misconceptions students have in learning C
  - Helps you practice for the final AND earn bonus (0.5% course grade)
  - Stay tuned! There will be a note on Piazza, which will be emailed out to everyone

Office Hours

- We’ll be holding additional extra office hours starting this Thursday. Stay tuned and check the course calendar.

Thursday,	12pm-1pm	Jonathan	ICCS 008
Thursday,	3:30pm-4:30on	Michael	ICCS X237
Friday,	1:00pm-2:00pm	Hassan	ICCS 241
Monday,	2:00pm-4:00pm	Hassan	ICCS 241
Tuesday,	10:00am-12pm	Sean	ICCS X237
Tuesday,	4:00pm-6:00pm	Sean	ICCS X237
Wednesday	12:00pm-1:00pm	Michael	ICCS X237

Double Hashing, $f(i) = i \cdot hash2(x)$

- Probe sequence is
  - $h_1(k) \bmod size$
  - $(h_1(k) + 1 \cdot h_2(x)) \bmod size$
  - $(h_1(k) + 2 \cdot h_2(x)) \bmod size$
  - …

A Good Double Hash Function…

- is quick to evaluate.
- differs from the original hash function.
- never evaluates to 0 (mod size).

- One good choice is to choose a prime R < size
  - $hash_2(x) = R - (x \bmod R)$

Clicker Question

- Using the hash functions $h_1(x) = x \% 7$ and $h_2(x)=5 - (x \% 5)$ insert the following values using **double hashing** *76, 93, 40, 47, 10, 55*

- *In what index would would 55 be stored?*

- A: 6
- B: 0
- C: 1
- D: 3
- E: None of the above

Double Hashing Example

- Using the hash functions $h_1(x) = x \% 7$ and $h_2(x) = 5 - (x \% 5)$ insert the following values using **double hashing** *76, 93, 40, 47, 10, 55*

	insert(76) *76%7 = 6*	*insert(93)* *93%7 = 2*	*insert(40)* *40%7 = 5*	*insert(47)* *47%7 = 5* *5 - (47%5) = 3*	*insert(10)* *10%7 = 3*	*insert(55)* *55%7 = 6* *5 - (55%5) = 5*
0						
1				47	47	47
2		93	93	93	93	93
3					10	10
4						55
5			40	40	40	40
6	76	76	76	76	76	76
probes:	*1*	*1*	*1*	*2*	*1*	*2*

Clicker question

The primary hash function is: $\mathbf{h_1(k) = (2k + 5) \bmod 11}$.
The secondary hash function is: $\mathbf{h_2(k) = 7 - (k \bmod 7)}$

Hash these keys, in this order: *12, 44, 13, 88, 23, 94, 11*.
Which cell in the array does key 11 hash to?

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

A. 0
B. 2
C. 3
D. 4
E. 10

Clicker question (answer)

$h_1(k) = (2k + 5) \bmod 11.$ $\quad h_2(k) = 7 - (k \bmod 7)$

12, 44, 13, 88, 23, 94, 11. Which cell in the array does key 11 hash to?

$h(12) = (2(12) + 5) \% 11 = 7$

$h(44) = (2(44) + 5) \% 11 = 5$

$h(13) = (2(13) + 5) \% 11 = 9$

$h(88) = (2(88) + 5) \% 11 = 5 + 7 - 88\%7 = 8$

$h(23) = (2(23) + 5) \% 11 = 7 + 7 - 23\%7 = 12$

$h(94) = (2(94) + 5) \% 11 = 6$

$h(11) = (2(11) + 5) \% 11 = 5 + 2(7 - 11\%7) = 11$

A. 0
B. 2
C. 3
D. 4
E. 10

0	*11*
1	*23*
2	
3	
4	
5	*44*
6	*94*
7	*12*
8	*88*
9	*13*
10	

Load Factor in Double Hashing

- For *any* $\lambda < 1$, double hashing will find an empty slot (given appropriate table size and $hash_2$)
- Search cost appears to approach optimal (random hash):
  - successful search: $\frac{1}{\lambda}\ln\frac{1}{1-\lambda}$
  - unsuccessful search: $\frac{1}{1-\lambda}$

	λ=0.25	λ=0.5	λ=0.75	λ=0.9
Avg # slots searched	1.5	1.4	1.8	2.6

- No primary clustering and no secondary clustering
- One extra hash calculation

	λ=0.25	λ=0.5	λ=0.75	λ=0.9
Avg # slots searched	1.3	2	4	10

Deleting when using probing

Example:

- Suppose locations [97] to [101] are occupied in our hash table:

[0]	[1]	…	[97]	[98]	[99]	[100]	[101]	[102]	…
			dog	cat	goat	owl	deer		

- Suppose a new key hashes to [97]. Assuming a linear collision resolution policy, the key goes to 102.

- Later, suppose we delete the key that was hashed to [98].

- Add a tombstone (i.e., flag, marker) for 2 reasons:
  1. If searching, keep going when you hit a tombstone.

  2. If inserting, stop and add the item here.

This means that table entries can be occupied, deleted, or free

The Squished Pigeon Principle

- An insert using open addressing *cannot* work with a load factor of 1 or more.
- An insert using open addressing with quadratic probing may not work with a load factor of ½ or more.
- Whether you use chaining or open addressing, large load factors lead to poor performance!
- How can we relieve the pressure on the pigeons?

Hint: think resizable arrays!

Rehashing

- When the load factor gets “too large” (over a constant threshold on λ), rehash all the elements into a new, larger table:
  - takes O(n), but amortized O(1) as long as we (just about) double table size on the resize
  - spreads keys back out, may drastically improve performance
  - gives us a chance to retune parameterized hash functions
  - avoids failure for open addressing techniques
  - allows arbitrarily large tables starting from a small table
  - clears out lazily deleted items

Application: The 2-Sum Problem

- Given: Unsorted array of integers `A`, and a target sum `t`
- Goal: Determine whether or not there are two numbers `x` and `y` in `A` such that `x+y=t`

- Naïve solution: $O(n^2)$ exhaustive search
- Better solution:
  - Sort `A` $O(n \lg n)$
  - For each `x` in A look for `t-x` $O(n \lg n)$
- Amazing solution:
  - Insert elements of `A` into hash table `H` $O(n)$
  - For each `x` in `A`, lookup `t-x` in `H` $O(n)$

Application: De-Duplication

- Given a "stream" of objects
  - Linear scan through a huge file
  - Objects arriving in real time
- Goal: Remove duplicates (keep track of unique objects)
  - Report unique visitors to a web site
  - Avoid duplicates in search results
- Solution: When new object x arrives, look up h(x) and if not found insert.

The Pigeonhole Principle (Full Glory)

- Let X and Y be finite sets with $|X| = n$, $|Y| = m$, and $k = \lceil n/m \rceil$.

If $f : X \to Y$, then $\exists$ k values $x_1, x_2, \ldots, x_k \in X$ such that $f(x_1) = f(x_2) = \ldots f(x_k)$.

Informally: If n pigeons fly into m holes, at least 1 hole contains at least $k = \lceil n/m \rceil$ pigeons.

Pathological Data Sets

- For good hash performance, we need a good hash function
  - Spreads data evenly across buckets
- Ideal: Use super-clever hash function guaranteed to spread every data set out evenly
- Problem: Such a hash function does not exist
  - For every hash function, there is a pathological data set

Pathological Data Sets

- Reason
  - Fix a hash function h
  - Let U be the potential number of keys
  - Let m be the table size
- There exists an array cell i, such that at least U/m elements hash to i under h
- If data set drawn only from these elements, then everything collides.
- This data set could be quite large since $U >> m$

Overview of Universal Hashing

- For every deterministic hash function, there is a pathological data set.
  - Solution: Do not commit to a specific hash function

- Use randomization
  - Design a family H of hash functions, such that for every data set S, most functions $h \in H$ spread S out "pretty evenly"

Review question from last year's midterm

What is printed to the console when magic(5) is called?

```
#define MAX_VAL 150

void magic(int n)
{
    if( n  <= 0)
      return;
    if( n > MAX_VAL)
      return;
     printf("%d ",n);
     magic( 2*n);
     printf("%d ",n);
     return;
}
```

5 10 20 40 80 80 40 20 10 5

Learning Goals revisited

After this unit, you should be able to:

- Define various forms of the pigeonhole principle; recognize and solve the specific types of counting and hashing problems to which they apply.
- Provide examples of the types of problems that can benefit from a hash data structure.
- Compare and contrast open addressing and chaining.
- Evaluate collision resolution policies.
- Describe the conditions under which hashing can degenerate from O(1) expected complexity to O(n).
- Identify the types of search problems that do not benefit from hashing (e.g. range searching) and explain why.
- Manipulate data in hash structures both irrespective of implementation and also within a given implementation.